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ANTENNA AND RECEIVING-SYSTEM NOISE-TEMPERATURE CALCULATION

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ABSTRACT

In Part I, a calculated curve representing the noise temperature of a typical directive antenna in the frequency range 100 to 10,000 Mc is presented, together with the method and details of calculation. Representative environmental conditions and antenna pattern characteristics are assumed. Since antenna noise temperature averaged over all galactic directions is not directly affected by the antenna gain and beamwidth, this curve may be used as an approximation for any directive antenna in this frequency range. The values given by this curve may be readily modified for other assumed or actual conditions.

Part II presents a methodology for utilizing this antenna noise temperature in calculation of a receiving-system noise temperature, from which the total system noise power output and the signal-to-noise power ratio may conveniently be computed. Basic concepts and definitions are first reviewed and then applied to development of equations for the noise temperatures of system components and an overall system of cascaded components, referred to an arbitrary point within the system.

The need for definition of both the spot (frequency-dependent) noise temperature and the average temperature over a passband is pointed out, and also the need for definition of a transducer noise temperature that represents only the intrinsic transducer noise. The IRE-defined input noise temperature of a twoport transducer is interpreted to include, in the case of a multiple-input-response transducer, the effect of noise power contributed by a standard-temperature input termination via the spurious responses. For the purposes of this report, a quantity called "principal-response effective input noise temperature" is defined. It is equivalent to the IRE-defined temperature with the contribution of the input termination (via the spurious responses) deleted.

The use of system noise temperature for comparing the low-noise merit of different systems is discussed. It is pointed out that for this purpose the system temperature must be referred to the system input terminals, and that these terminals must be defined to precede all system elements that result in dissipative loss, including loss that may occur in the antenna structure. Moreover, if the antenna is included as part of a system being thus rated, some standard or convention as to the noise environment (such as the assumptions made in calculating the curve in Part I) is needed.

ABSTRACT (Continued)

The calculation of received signal power for various types of systems (one-way radio, monostatic and bistatic radar, satellite-reflection communication) is briefly reviewed, to show how the system noise temperature may be used for signal-to-noise-ratio calculation. The case in which signal power may be simultaneously received via more than one input response channel of a multiple-response receiver (as in radiometry) is briefly considered.

The report is basically oriented to the problem of radar maximum range calculation, but has application to radio receiving systems in general.

PROBLEM STATUS

The work described in this report is a part of a more comprehensive and continuing project. This is a final report on this phase of the project.

AUTHORIZATION

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ANTENNA AND RECEIVING-SYSTEM NOISE-TEMPERATURE CALCULATION

FOREWORD

The procedure to be described is applicable to both radar and general radio receiving-system noise-temperature calculation. The immediate application for which it was devised is that of radar maximum range calculation. The development of a standardized procedure for radar range calculation was undertaken some time ago at the Naval Research Laboratory, partly to meet the Laboratory's own needs, and partly because there seemed to be a need for a method that could be used as a standard by contractors submitting radar system proposals to the Navy Bureaus. The work has been encouraged and partially sponsored by the Bureau of Ships.

In preliminary reports of this undertaking (1,2), an antenna temperature curve was offered for use in general radar range calculation, in conjunction with an equation for a system noise temperature, but without detailed explanation. This report provides the explanation, together with additional details for calculating the system noise temperature of relatively complicated systems. This is the third in a series of reports on details of the range calculation procedure presented in Refs. 1 and 2. The first two dealt with tropospheric absorption loss (3) and tropospheric refraction (4).

Part I considers the problem of antenna noise-temperature calculation, and gives the method and assumptions for calculating an antenna temperature curve as a function of the radio frequency and the antenna beam elevation angle. The curve thus calculated may be used in radio or radar system performance calculation. Part II discusses the principles and procedure for utilizing the antenna noise temperature value (whether obtained from a standard curve or otherwise determined) in an evaluation of the total system noise power (system noise temperature), and of the signal-to-noise ratio.

As the references indicate, much has been written by others on the subject of noise temperature. The principal purpose of the present report is to justify in some detail the noise-temperature aspect of a previously published procedure for radar range calculation. It seemed necessary because there has been some confusion concerning noise temperature principles and definitions. Although some excellent papers have been written to clarify specific aspects of the subject, there has not been one, to the author's knowledge, which would serve the present purpose. The aim is primarily to give a coherent formulation of already established ideas, adhering as closely as possible to accepted definitions and terminology.

In a few cases, however, it has seemed necessary to employ definitions, notation, and terminology that are not part of the standard or accepted usage. For example, in the IRE standards, the distinction between spot and average noise factors has not been carried over into the realm of noise-temperature definition, at least not explicitly, but in writing this report it became evident that a similar differentiation must be made for noise temperature, and this has been done. The terms spot-frequency noise temperature and average noise temperature have been used, although this is not a customary practice in the literature. Possibly some better terminology could be devised.

Also, a departure has been made from the standard definition of transducer (receiver) input noise temperature. There is some confusion about the present IRE definition as it applies to a multiple-response (heterodyne) transducer; the interpretation made here, on the basis of its defined relationship to the IRE noise factor, assumes that the IRE definition expresses the noise contribution of a standard-temperature input termination via the spurious responses, in addition to the intrinsic transducer noise. This definition is not suitable for system noise-temperature calculation in the multiple-response case, and the redefinition made for the purposes of this report expresses only the intrinsic transducer noise; it is equivalent to the IRE-defined temperature with the spurious response input-termination-noise-temperature contribution deleted.

As these comments imply, the noise-temperature method of calculating and representing the noise performance of receiving systems has not yet been fully developed and standardized. It is hoped that this standardization may soon be accomplished, even though some parts of the present report might thereby become outmoded.*

In the matter of notation, apology is offered for using the symbol $\overline{NF}$ for the noise factor, instead of the IRE-approved F. This was done because F had already been assigned to the pattern-propagation factor in the radar equation, in previous reports of this series (1,2). The notation T_N has been used for the system noise temperature, instead of T_S, because T_S is used to denote the noise temperature of the sun.

*When this report was almost completed, a paper which parallels some of the ideas of Part II was presented by a panel at the IRE PGMTT 1961 National Symposium. This paper, listed as Ref. 28, also attempts an integration of noise-temperature concepts from the system point of view. There are however some differences in the approach of the PGMTT panel, especially in the bandwidth referencing of their system noise temperature (which they call system operating noise temperature, T_{op}) and of their effective input noise temperature of a multiple-response transducer (which they call broad-band effective input noise temperature, T_b). (The analogous quantities of the present report are system noise temperature, T_N, and principal-response input noise temperature, T_p.) Despite these differences, the basic system-noise-temperature philosophies are the same. The appearance of the PGMTT panel's paper stimulated a more thorough treatment of the problem of multiple-response receivers in this report. (Originally, only the case of a single image response had been treated.) Also, discussion with one of the panel members, M.T. Lebenbaum of Airborne Instruments Laboratory, helped to clarify aspects of the noise-temperature noise-factor relationship for multiple-response systems, as expressed by Eqs. (32), (33), (35), and (36); however, responsibility for the correctness of these equations, which may be controversial, rests with the author. The reasoning on which these equations are based is given in Appendix A.

PART I - ANTENNA NOISE TEMPERATURE

INTRODUCTION

The noise received by a radio or radar system from natural external radiating sources has become of increasing practical importance in recent years, as receivers have been improved and internal noise has been minimized. The external noise now often sets the limit on system performance, even at the microwave frequencies.

Since the discovery of cosmic noise by Jansky in 1932, many studies of this external noise have been made, resulting in data on which engineers may base estimates of system performance. However, the evaluation of all the contributing factors in a specific case is a difficult and complicated task. Often, engineers wish to make a system performance calculation which takes the "antenna noise" into account in a general way, without the necessity of a detailed investigation. The curve presented in this report (Fig. 1) allows this to be done. For many applications the accuracy provided by this curve will be adequate. In cases where better accuracy is needed, simple modification of the values given by the curve may be made, to allow for conditions different from those assumed in computing it. In special cases, complete calculation of the antenna temperature may be required, using methods similar to that used for calculating the curve. In still other cases, measurement of the antenna temperature may be feasible. These procedures are stated in increasing order of difficulty and reliability of the final result. Use of a curve such as Fig. 1 is an approximation procedure, but it is a considerable improvement compared to ignoring antenna noise, or assuming a constant value at all frequencies.

The curve assumes a surface-based antenna — that is, one within perhaps a few hundred feet of the earth's surface. The principal effect of this assumption is that the entire earth's atmosphere is interposed between the antenna and extraterrestrial noise sources. The amount of noise received from ground radiation is also affected. The curve is not applicable to airborne radar antennas nor to ground-based antennas at extremely high altitudes, although the error incurred by using it in these cases would be serious only at the microwave frequencies. The ensuing discussion will provide a basis for making the corrections that would be necessary in the case of a high-altitude antenna.

The frequency limits of the curve were chosen to correspond to the frequency region of primary interest for long-range search radar. However, both lower and higher frequencies are of use in radar as well as other applications. The curve could be extended in both directions by application of the principles to be discussed. At frequencies below 100 Mc, however, the ionosphere plays an increasing role, both as an absorber of cosmic noise and as a generator of noise. The great variability of the ionosphere's characteristics, especially diurnally, makes a single curve for antenna temperature a somewhat dubious concept in this frequency region. At frequencies above 10,000 Mc, the great variability of the water-vapor content of the lower troposphere, on which the tropospheric absorption and noise generation strongly depend, imposes a similar questionableness on the value of a single curve.

Definition of Antenna Noise Temperature

It has become the accepted practice to represent the noise power received by an antenna from permanent external radiating sources by an effective antenna noise temperature, T_a. The noise from these permanent sources is similar in character to thermal

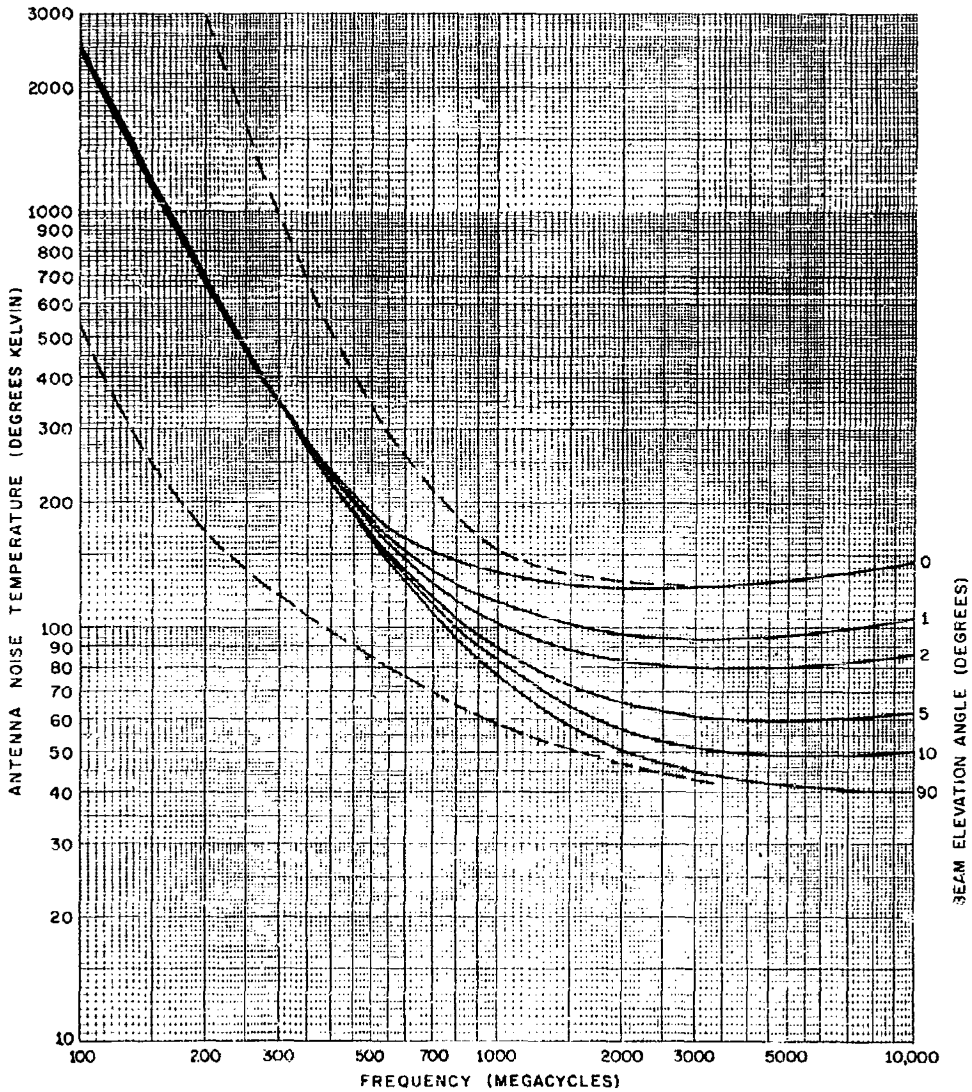

Fig. 1 - Noise temperature of a typical directive antenna for representative environmental conditions

noise, and combines with the internal receiver noise in a simple additive manner. The word "permanent" here implies that temporary or sporadic sources of noise (e.g., jamming signals and other man-made noise) are not considered, and are ordinarily not treated by the antenna-temperature method.

The antenna noise temperature is a fictitious absolute (Kelvin) temperature such that the received noise power per unit bandwidth (spectral power density) is

$$S_a = k\ T_a \tag{1}$$

where k is Boltzmann's constant (1.38 x 10^{-23} watt-seconds per degree). The power in a bandwidth B is then $k\ T_a\ B$. S_a is the "available" power density, or that which would be delivered to a matched load, as discussed in Part II. The theoretical basis of the antenna-temperature concept is given in radio astronomy texts, such as that of Pawsey and Bracewell (5).

Receiving-System Noise Temperature

Part II deals with the general problem of computing the total noise power and signal-to-noise ratio, employing the concept of a system noise temperature. Here the general procedure will be outlined very briefly, as background for the discussion of antenna noise temperature.

The total noise power in the receiving system may be represented by a system noise temperature, T_N, such that the total available noise power referred to the receiver input terminals is $k\ T_N\ B_N$, where B_N is the receiver noise bandwidth. The referral concept is discussed in Part II. As shown there, the system noise temperature may be referred to any point in the system with equal usefulness for output signal-to-noise-ratio calculation. For some purposes the receiver input is not an appropriate reference point, but it is a customary one and is assumed in this part of the report.

Generally, T_N may be regarded as the sum of three components: (a) the contribution of the antenna due to reception of noise from external radiating sources, (b) the thermal noise generated due to dissipative losses in the receiving transmission-line system, and (c) the noise from sources internal to the receiver itself. Each of the latter two contributors is assigned an effective noise temperature, called respectively receiving-transmission-line output noise temperature, T_r, and effective receiver input noise temperature, T_e.

This division of the receiving system into three components, shown schematically in Fig. 2, is arbitrary. For example, any point in the system could be called the receiver input terminals, and often the antenna system contains transmission line. Thus it is quite possible, by definition, to eliminate the second component, the transmission line. But generally, in practical systems, it is identifiable as a separate entity, and it is so treated here. As a matter of fact, for a reason discussed in detail in Part II, the transmission line is here regarded as beginning at the reception surfaces of the antenna, so that any dissipative losses in what could be considered the antenna are called transmission-line losses.

The power loss factor* of the transmission-line system, L_r, figures importantly in the system noise-temperature calculation. First, this loss acts to reduce the amount of antenna noise power reaching the receiver input terminals. Hence the additive contribution of the antenna temperature to the system noise temperature is T_a/L_r, rather than T_a directly. Second, the magnitude of the transmission-line loss factor directly affects the transmission-line noise temperature, T_r.

*Power loss factor is defined as the ratio of power input to power output of the lossy device or component.

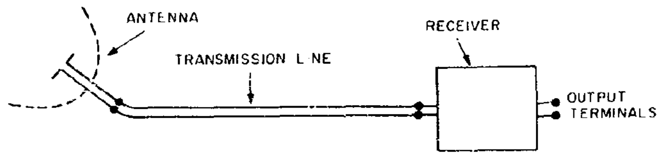

Fig. 2 - Components of a basic receiving system

The formula for the system noise temperature T_N of a single-response system* is

$$T_N = T_a/L_r + T_r + T_e. \tag{2}$$

The formula for the transmission-line output noise temperature T_r is

$$T_r = T_t\ (1 - 1/L_r) \tag{3}$$

where T_t is the thermodynamic (thermal) temperature of the lossy elements of the line. The effective input noise temperature of the receiver, T_e, is defined in terms of the receiver noise factor, $\overline{NF}$:

$$T_e = (\overline{NF} - 1)\ T_0 \tag{4}$$

where $T_0 = 290°K$ is the reference temperature for receiver noise factor measurement. The theoretical basis of these equations is given in Part II, together with more precise definitions of the quantities involved.

The quantities L_r, T_t, and $\overline{NF}$ or T_e may in general be measured or estimated for a particular system. It then remains to evaluate T_a.

Concept of an Antenna-Temperature Curve

The effective antenna noise temperature, T_a, may be computed for a particular antenna in a particular environment by well-known methods, but the exact procedure is quite complicated and tedious. Fortunately, if an antenna has a unidirectional pattern that is not extremely broad, as do the majority of antennas above 100 Mc, the beamwidth and gain have little or no effect on its noise temperature (averaged over all galactic directions) under ordinary circumstances. Therefore it is possible to calculate an antenna noise temperature, as a function of frequency, that is approximately applicable to any typical directional radar or radio antenna.

In the uhf region and above, it is necessary to introduce an additional dependence upon the beam elevation angle. In this frequency region the antenna temperature is partly due to thermal noise generated in the absorptive gases of the atmosphere. The absorption, and hence the noise power, is dependent upon the thickness of the atmospheric path traversed by the antenna beam, and hence upon the beam elevation angle. It is for this reason that the curve in Fig. 1 diverges into several branches, corresponding to specific elevation angles, above about 400 Mc.

Antenna noise temperature curves generally resembling the one presented here have been published by several other authors (6-8), but these have been primarily designed to show the general trend of the temperature with frequency, to aid in the selection of an optimum frequency of operation for specific purposes. They are not adapted to numerical use in radar or radio system performance calculation, the purpose for which Fig. 1 is intended.

*Methods of calculating the noise temperature of a multiple-response system are given in Part II. The definition of receiver input temperature given by Eq. (4) is not applicable to Eq. (2) in the case of a multiple-response system, and modification of the first two terms must also be made.

For a great many performance-calculation purposes, it is not as important to obtain a result that is applicable to a particular operating site at a particular time as it is to employ some reasonable assumption concerning the noise environment that has been adopted as standard. (A similar statement is true of many of the other environmental conditions, some of which are also discussed in previous reports of this series (1-4).) The curve presented here may serve as an interim standard for such purposes. Ultimately it may prove desirable to have official noise-environment standards, based on similar though not necessarily identical assumptions as those made for Fig. 1, adopted by some appropriate agency or professional organization.

In the ensuing pages, much information that is available elsewhere will be presented. This is done to establish clearly the basis on which the antenna temperature curve was computed. The sources of the information are given specifically wherever practicable. The list of references at the end of the report gives all the sources of material utilized. In general, the detailed theoretical derivation of equations is not given in Part I, since they are available in the references (5,9-13). However, Part II does present detailed analysis of a few matters on which the author has been unable to find satisfactorily detailed explanations elsewhere.

EQUATIONS AND DATA FOR ANTENNA-TEMPERATURE CALCULATION

In the expression for the effective system noise temperature, T_N, Eq. (2), the quantity T_a is the only one not determinable by measurements made on components of the radar or radio receiving system in the laboratory. It is basically determined by the environment rather than by the system, although for a given environment it is possible to minimize T_a by minimizing antenna side lobes and back lobes. Here the assumption of typical side- and back-lobe levels is made.

The following outline of the theory of calculating antenna noise temperature is cursory. The reader unacquainted with the subject who wishes a more detailed treatment will find it in any of several references at the end of this report. The purpose here is to show very briefly how the working formulas used in calculating Fig. 1 were developed.

The effective antenna noise temperature is the composite result of all the natural radiating noise sources within the total pattern of the antenna, which in general extends over 4π steradians even though the main beam may be confined to a small part of this solid angle. These sources include, usually, the cosmos, the sun, the ionosphere, the troposphere, and the ground or earth, which includes the sea as well as solid ground, and such structures as buildings and ships.

Each of these separate radiating sources has a "noise temperature" for which the notation T_C, T_S, T_G, etc., will be employed (where the subscripts stand for cosmic, solar, ground, etc.). This quantity is equal to the antenna noise temperature that would result if the source were viewed by an "ideal" antenna having a power pattern confined solely to this source.

But practically, antennas have patterns that include side and back lobes, and the main beam itself may be only partially "filled" by a particular source. The result is that each source contributes to the effective antenna noise temperature something less than its own effective noise temperature. If in general there are n sources contributing to the total antenna noise temperature, the contribution of the i^{th} source will be designated $T_{a(i)}$, the total being the sum of the n contributions.

If a cosmic source is viewed through a lossy (absorptive) propagation medium, the propagation loss attenuates the noise from the cosmic source, but adds a further thermal noise contribution of its own, proportional to the fractional absorption and to the absolute thermal temperature of the medium. If the medium is at a uniform thermal temperature, its thermal noise contribution, expressed as an effective noise temperature, is given by the same formula that applies to a lossy transmission line, Eq. (2). (The fractional absorption is $1-1/L$, where L is the power loss factor.)

Generally a cosmic source, although actually of great depth in space, may be treated as if it were a radiating <u>surface</u> characterized by a noise temperature T. In general T may vary from point to point over this surface, and from the viewpoint of the earth-located antenna T is a function of angular direction, as are also the antenna power-gain function, G, and the propagation-medium loss factor, L. If such a source occupies a solid angle Ω_i within the power pattern of the antenna, the contribution to the total antenna temperature from within Ω_i is

$$T_{a(i)} = \frac{1}{4\pi} \int_{\Omega_i} G\left[\frac{T}{L} + \left(1 - \frac{1}{L}\right) T_t\right] d\Omega \tag{5}$$

where the first term inside the brackets is representative of the contribution of the cosmic source itself and the second term represents the noise contribution of the propagation medium whose loss factor is L and thermal temperature T_t. Actually this second term is an approximation, or at any rate it is an abbreviated expression. The thermal temperature and the absorption of the medium generally vary over the propagation path, and a complete formula would take this into account. However, the formula given here is correct if T_t is interpreted as a weighted average thermal temperature. In practice, since most of the absorption occurs in the lower atmosphere, and the temperature varies with time anyway, it is usually sufficiently accurate to assign the value 290°K (16.8°C, 62.3°F) to T_t.

Equation (5) may also be applied to radiation from the earth (ground). In this case it may be assumed that $L = 1$, so that the second term of the equation disappears.

The effective noise temperature of a material substance is proportional to its thermal temperature and to its capacity for absorbing radio waves. Thus, the earth may be at an approximate thermal temperature of 290°K, but if, for example, its reflection coefficient is of magnitude $\rho = 0.8$, the effective noise temperature is 0.2 x 290 = 58°K.

When a surface, such as the earth, partially reflects radio waves, as in the above example, there is a further contribution to the antenna noise temperature from any source that may be reflected to the antenna by the surface. Thus if the source seen by reflection ordinarily has a noise temperature T_1, the effective noise temperature seen by reflection, from the surface of reflection coefficient $\rho = 0.8$ (for example), would be $(0.8\,T_1)$; this would be the value used as T in Eq. (5) for computing the contribution of the reflected source.

If a noise source is seen simultaneously by direct and reflected propagation paths, the coherence (phase relation) of the noise signals received by the two paths must in principle be taken into account. This will in general best be accomplished by substituting for the free-space antenna power pattern the reflection lobe-structure, as computed by well-known methods, translated into an equivalent antenna power pattern through the relation $G(\theta,\phi) = G_{max} F^2(\theta, \phi)$, where G_{max} is the (free-space) antenna gain in the beam maximum, and F is the pattern-propagation factor as defined by Kerr (14). This effective pattern is used to determine G in Eq. (5).

The pattern-propagation factor, F, is defined by assuming the antenna to be radiating instead of receiving. F is the ratio of the field strength (electric intensity) observed in the direction (θ,ϕ) at a distant point to that which would be observed at the same point if only the direct propagation path existed and if the maximum-gain portion of the antenna pattern were aimed in this direction. For details and methods of computing F, see Kerr (14).

Ordinarily, an analysis in this much detail is not required, and approximation techniques may be applied. Approximation methods are especially justified when the quantity being computed is subject to appreciable random variation, so that any computed result is statistical in nature, as is the antenna noise temperature.

The principal approximation that will be made is to assume that T and L are constants over the solid angle Ω_i. Then by application of the Theorem of the Mean of integral calculus, Eq. (5) may be written

$$T_{a(i)} = \frac{\Omega_i \bar{G}}{4\pi}\left[\frac{T}{L} + \left(1 - \frac{1}{L}\right)T_t\right] \tag{6}$$

where $\bar{G}$ is the average power gain of the antenna over the solid angle Ω_i. This result may be condensed to

$$T_{a(i)} = \frac{\alpha_i T}{L} + \alpha_i T_M \tag{7}$$

where $\alpha_i = \Omega_i \bar{G}/4\pi$, in terms of a transmitting antenna, is the fraction of the total radiated power radiated into the solid angle Ω_i, and where T_M is the effective noise temperature of the propagation medium.

The principle of Eq. (7) may be used when several sources are present, partially overlapping in angle, and when the propagation path is separable into distinctly different segments, such as the troposphere and the ionosphere, to obtain an expression for the total antenna temperature. In terms of the usual contributing sources this expression is

$$T_a = \frac{\alpha_C T_C}{L_I L_T} + \frac{\alpha_S T_S}{L_I L_T} + \frac{\alpha_I T_I}{L_T} + \alpha_T T_T + \alpha_G T_G \tag{8}$$

where the α's are as just defined, for each source, and the other symbols have the following meanings:

T_C - effective noise temperature of space (cosmos, galaxy)

T_S - noise temperature of the sun

T_I - noise temperature of the ionosphere

T_T - noise temperature of the troposphere

T_G - noise temperature of the ground

L_I - power loss factor for one-way propagation through the ionosphere (ratio of incident to emergent power)

L_T - power loss factor for the troposphere.

The terms "sky noise" and "sky temperature" are sometimes used to refer to the complex of noise sources comprising the cosmos, sun, ionosphere, and troposphere. In the antenna-noise-temperature calculations shown later in this report, ionospheric loss and noise temperature are ignored since they are negligible in the frequency range considered, but their effect is included in Eq. (8) for completeness.

In an equation of this type, if there are n individual sources occupying solid angle sectors that do not overlap, and if altogether they occupy the total solid angle 4π steradians, then

$$\sum_{i=1}^{n} \alpha_i = 1 . \tag{9}$$

However, in the example given it is evident that the α's do not fulfill this condition, because, for instance, α_C and α_S apply to solid angle sectors that overlap, and α_C, α_I, and α_T apply to essentially one and the same sector. Nevertheless, Eq. (9) is useful as a general principle for guidance in evaluation of some of the α's.

In the following sections, evaluation of the factors for the individual noise sources of Eq. (8) will be discussed, and then an example of antenna noise-temperature calculation will be given.

Cosmic Noise

Radiation from outer space, called cosmic noise and sometimes galactic background noise, is radiated by the hot gases of stars and by matter distributed in interstellar space. It is most intense in the galactic plane and reaches a maximum in the direction of the galactic center. In some parts of the sky it is very low. (It is customary to speak of "hot" and "cold" parts of the sky.) Cosmic noise is a function of frequency, decreasing with increasing frequency. It is likely to be a major contributor to the total system noise in the vhf region (below 300 Mc) and is usually a minor factor in the microwave region (above 1000 Mc). A paper by H. C. Ko (15) presents a collection of data, including maps of sky temperature due to cosmic noise at various frequencies.

As previously indicated, an ideal narrow-beam antenna— one whose radiation and reception pattern consisted entirely of a main beam, with no side-lobe or back-lobe pattern components — when pointed at a portion of the sky of uniform noise temperature, would have an effective antenna noise temperature equal to the sky temperature if there were no other noise sources. The antenna gain and beamwidth do not directly have any effect. However, the hottest parts of the sky, the galactic center and galactic plane, are confined to a belt about 3 degrees wide, so that the maximum values of antenna temperature are obtained when a beam less than 3 degrees wide is pointed at the galactic center. If the antenna beamwidth is greater than 3 degrees, the beamwidth has a marked effect on antenna temperature in this region. Stronger-than-average noise radiation is received from a broader region within about 30 degrees from the galactic plane. Then there is a component of radiation that is roughly constant in all directions. The strength of this minimum (isotropic) component is so low above about 200 Mc that it is difficult to measure it.

Antenna temperatures due to cosmic noise may be as great as 10,000 degrees or more for a narrow-beam antenna pointed at the galactic center in the region of 100 Mc, and less than one degree above about 1000 Mc for an antenna pointed at a "cold" part of the sky. However, at these higher frequencies, such low antenna temperatures cannot actually be realized because of noise radiation from the earth's atmosphere, the ground, and the sun.

Generally, in calculating radar or radio system performance, it is not possible to predict the exact part of the cosmos at which the antenna will be pointing. Therefore, maximum, average, and minimum values of cosmic noise are of interest. Hogg and Mumford (8) give the following approximate formula for the average cosmic noise temperature:

$$T_{C(\text{average})} = 290\,\lambda^2 = \frac{2.6 \times 10^7}{f^2} \tag{10}$$

where T_C is the Kelvin temperature, λ is wavelength in meters, and f is frequency in megacycles. The actual law that cosmic temperature follows is more like $T_C \propto f^{-2.5}$, but the exact exponent is not known, partly because of the difficulty of precise measurement and partly because it varies for different parts of the celestial sphere. Therefore Eq. (10) is sufficiently accurate for many purposes, and was used in computing Fig. 1.

Brown and Hazard (16) have given estimates of maximum and minimum cosmic temperatures as a function of frequency. The dashed curves of Fig. 1 are based on their estimates. The maximum for 100 Mc, which is not shown because it exceeds the range of temperatures conveniently accommodated on the graph, is 17,400°K. (This is not the maximum cosmic temperature, but the resulting antenna temperature when the cosmic temperature is maximum.)

The fraction α_C for the cosmos will usually consist primarily of the integrated power pattern of the main beam above the horizon, plus portions of the main beam reflected upward by reflecting surfaces of the earth or sea, plus the side and back lobes directed or reflected upward. Hansen (17) has calculated that for a paraboloidal-reflector antenna the fraction of power in the main beam varies from 0.83 to 0.98 depending on the illumination taper, "spillover," reflector leakage, etc., the 0.98 figure being for an illumination taper giving -25 db side lobes, assuming no spillover or leakage. Taking spillover and leakage into account, by assuming an aperture efficiency of 0.65, he concludes that $\alpha = 0.85$ is a representative value for the main beam of practical antennas.

This means that the total fraction of power in the side lobes is 0.15. Since about half of these are directed upward, and still others are reflected upward, in the calculations leading to Fig. 1 it is assumed that $\alpha_C = 0.95$. This fraction is assumed even at zero elevation angle of the main beam because, at least in many typical cases, most of the main beam will be reflected upward.

Solar Noise

An important additional source of antenna noise at some frequencies is the sun. Generally, it is not necessary to point an antenna directly at the sun, but because the sun is such a powerful noise source, during sunspot activity it can contribute appreciably to antenna noise through side lobes of the antenna pattern.

The noise temperature of the quiet sun, at frequencies needed for plotting Fig. 1, are given in Table 1. These values are attributed by Matt and Jacomini (6) to a summary made by Van De Hulst (18) of the results of various observers.

During sunspot activity, levels from 10^2 to 10^4 greater than those given by this table are observed for periods of seconds (solar bursts), followed by levels about 10 times the quiet level lasting several hours.

Table 1
Noise Temperature of the Quiet Sun (after Van De Hulst; values read from curve given by Matt and Jacomini)

Frequency (Mc)	Noise Temperature (°K)
100	1×10^6
200	9×10^5
300	7×10^5
600	4.6×10^5
1000	3.6×10^5
3000	6.5×10^4
10,000	1.1×10^4

According to Matt and Jacomini, the values of Table 1 assume that the sun's "noise diameter" is 1/2 degree. Actually, the sun has an effective noise diameter that is greater than 1/2 degree at low frequencies, but the values of Table 1 are corrected for this effect so that they give correct results when applied in a formula of the type of Eq. (6) assuming Ω_i to be 6×10^{-5} steradians (the value corresponding to 1/2-degree diameter) at all frequencies. Therefore the antenna noise temperature contribution of the sun, the second term of Eq. (8), is

$$T_{a(s)} = 4.75 \times 10^{-6} \frac{\bar{G}\, T_s}{L} \tag{11}$$

where $\bar{G}$ is the average antenna gain over the sun's disk, assumed to be 1/2 degree in diameter, T_s is given by Table 1 for the quiet sun, and L is the absorption loss in the propagation path.

If the sun's noise temperature were uniform over the disk and if the noise diameter were exactly 1/2 degree at all frequencies, this formula would be applicable even to antennas of less than 1/2-degree beamwidth, because $\bar{G}$ becomes independent of the antenna beamwidth in this case. But since neither of these conditions is actually met, the formula is basically applicable to antennas of beamwidth greater than 1/2 degree.

In computing Fig. 1, a solar noise temperature ten times the quiet level was assumed, together with a value of $\bar{G} = 1$. This value of $\bar{G}$ is a reasonable side-lobe level for a well-designed directive antenna. If the main beam of an antenna is directed at the sun, the resulting antenna temperature will be high, but it is assumed here that ordinarily the sun will be in the side-lobe pattern.

At the unity-gain side-lobe level, during solar bursts the sun's contribution to antenna noise temperature may be as great as 1000°K at 1000 Mc, and 5000°K at 100 Mc. Cosmic noise temperatures at 1000 Mc are well below 1000°K, so that the effect of a solar burst would be noticeable at this frequency with a low-noise receiver. At 100 Mc, however, this solar-burst noise level is comparable to the average cosmic noise level, and so would not be as dramatically noticeable as at the higher frequencies.

Propagation-Medium Noise

The propagation medium contributes to antenna noise at frequencies at which the medium is absorptive. At the frequencies of interest here, only the troposphere has appreciable absorption, although at lower frequencies the ionosphere is absorptive and the troposphere is not. For this reason the effect of the ionosphere was included in Eq. (8), but in computing Fig. 1 no ionospheric effect was considered. Although there is some ionospheric absorption above 100 Mc, it is quite small according to Millman (19), and in the vhf region any small noise contribution from the ionosphere is negligible compared to cosmic noise, according to Little and Leinbach (20).

As mentioned in the general discussion of antenna-noise-temperature calculation leading to Eqs. (5), (6), and (7), the formula for the contribution of the propagation medium is similar to that of a lossy transmission line, Eq. (2). If the absorption of the troposphere is expressed by the loss factor L_T, and if the thermal temperature of the medium is T_t, then the antenna-noise-temperature contribution will be

$$T_{a(T)} = \alpha_T\, T_t\, (1 - 1/L_T) = \alpha_T\, T_T\,. \tag{12}$$

As also mentioned previously, this formula is correct only if T_t is a properly weighted average over the propagation path. The more complete equation, in terms of a varying temperature and absorption coefficient over the path, is given by Dicke (21). But, as mentioned by Strum (12), Eq. (12) is good enough for most purposes, and 290°K is a suitable value for T_t.

Table 2 lists values of L_T, expressed in decibels, calculated by Blake (3). (The values originally calculated were for two-way transit of the path, applicable to radar propagation; the values shown here are half of these two-way values. The values for 90 degrees elevation had not been calculated for the radar case, but were calculated for the present application using the same method.) Also listed are the corresponding values of T_T, calculated according to Eq. (12) assuming $T_t = 290°K$.

Table 2
One-Way Attenuation, L_T, Decibels, and Noise Temperature, T_T, for $T_t = 290°K$

Freq. (Mc)	Ray Elevation Angle:														
	0°		0.5°		1.0°		2.0°		5.0°		10°		90°	
	L_T (db)	T_T (°K)	L_T (db)	T_T (°K)	L_T (db)	T_T (°K)	L_T (db)	T_T (°K)	L_T (db)	T_T (°K)	L_T (db)	T_T (°K)	L_T (db)	T_T (°K)
100	0.12	8	0.09	6	0.08	5	0.07	5	0.04	3	0.02	1	0	0
200	0.32	21	0.27	17	0.23	15	0.17	11	0.10	7	0.05	3	0	0
300	0.55	35	0.45	32	0.38	24	0.28	18	0.15	10	0.08	5	0	0
600	1.0	60	0.85	52	0.65	40	0.47	30	0.24	15	0.13	8	0.02	1.5
1000	1.4	80	1.1	65	0.85	52	0.60	37	0.29	19	0.15	10	0.03	2
3000	1.7	94	1.3	75	1.0	60	0.70	43	0.33	21	0.17	11	0.04	2.5
10,000	2.3	119	1.7	94	1.3	75	0.90	54	0.44	28	0.22	14	0.05	3

These results are in approximate agreement with some calculations of D. C. Hogg (22), at the frequencies and angles for which comparison is possible. In the microwave region, Hogg's calculated temperatures and loss factors are slightly higher, probably due to assumption of slightly higher atmospheric water-vapor content (summer conditions).

To compute the antenna-noise contribution of the troposphere, $T_{a(T)}$, these tabulated values of T_T must be multiplied by α_T. In general α_T will be about the same as α_C, which has been estimated to be 0.95 for a typical practical antenna. However, the fact that T_T depends on elevation angle in a known manner should probably be taken into account here in at least an approximate manner. For example, if an antenna has a vertical beamwidth of 1 degree and is pointed up at 2 degrees elevation, the contribution to α_T of the main beam may be taken, as for α_C, to be 0.85. In the case of α_C, the additional fraction 0.1 was added to account for the side-lobe pattern which "sees" all other parts of the sky. But in the case of α_T, the other parts of the sky are at a different temperature except for the annular band at the same elevation angle. For zero elevation of the main beam the other parts of the troposphere are colder; for 90 degrees elevation they are hotter. Thus the side lobes have a greater effect on the antenna temperature when the beam is at a high angle. To give some weight to this effect without going through a rigorous analysis, the value $\alpha_T = (0.9 + 0.1 \sin \theta)$ was used for the computations. This formula gives for $\theta = 2°$, $\alpha_T = 0.9$; for $\theta = 5°$, $\alpha_T = 0.91$; for $\theta = 10°$, $\alpha_T = 0.92$, and for $\theta = 90°$, $\alpha_T = 1.0$.

Ground Noise

If part of the antenna pattern points toward the ground, which is at some physical temperature usually in the vicinity of T_0 (290°K), there will be a contribution to the antenna noise from this source because of so-called blackbody radiation. If the solid angle subtended by the ground visible from the antenna location is Ω_G steradians, and if the average gain of the antenna within this solid angle is $\bar{G}$, the antenna noise-temperature contribution from this source will be, on the basis of Eq. (6),

$$T_{a(G)} = \frac{\Omega_G T_G \bar{G}}{4\pi} = \alpha_G T_G \tag{13}$$

where T_G is the effective noise temperature of the ground. This will be approximately equal to its thermal temperature if the ground is lossy (absorbs radio waves). If the ground is totally reflecting (e.g., the sea, and sometimes the earth at small grazing angles), its noise temperature will be equal to that of the portion of sky reflected by it. If the reflection is specular, the effect is to set up an interference pattern as discussed following Eq. (5), and the effect on antenna noise should be analyzed in terms of this pattern in relation to cosmic sources, rather than in terms of an independent contribution from the reflecting ground surface. The factor L is omitted from Eq. (13) because ground sources are ordinarily so close to the antenna that no appreciable propagation losses occur.

In the typical case, the principal contribution to the term $\bar{G}$ will be the side-lobe and back-lobe pattern of the antenna. Dr. W. W. Ward, of MIT Lincoln Laboratory, has estimated* that $\bar{G}$ will range from about 0.1 (-10 db) to 0.5 (-3 db) over the entire region outside the main beam. Individual side lobes of antennas may have positive gains of several decibels, but typically they are of about unity gain (zero decibels) or less.

In computation of the antenna-temperature curve, a value $\alpha_G = 0.124$ was used, somewhat arbitrarily; this corresponds to $\Omega_G = \pi$ steradians and $\bar{G} = 0.5$, and for $T_G = 290°K$ results in $T_{a(G)} = 36°K$. This is an intermediate value in the range of about 20°K to 60°K which is believed to represent the ground contribution in current practical experience.† The ground temperature contribution in any actual case will vary greatly with the siting of the radar. With smooth sea or terrain and horizontal polarization, the ground temperature contribution should be quite low because the ground will act as a reflector rather than an absorber. Incidentally, the value $\alpha_G = 0.124$ is not completely consistent with the assumption $\alpha_C = 0.95$ for the sky, but may be justified as a compensation for the possibility that some parts of the ground are effectively hotter than 290°K, due to sun reflections. This effect may be especially likely with a rough sea.

Computation of Antenna-Temperature Curve

In application of the principles and assumptions described in the foregoing discussions of the contributing sources, the following specialization of Eq. (8) may be used to compute a typical antenna noise temperature curve, for various frequencies and beam elevation angles:

$$T_a = \frac{0.95\ T_C}{L_T} + \frac{4.75 \times 10^{-5}\ T_{SQ}}{L_T} + (0.9 + 0.1 \sin \theta)\ T_T + 36 \tag{14}$$

where T_C is given by Eq. (10), T_{SQ} is the effective noise temperature of the quiet sun, from Table 1, and L_T and T_T are tabulated as functions of frequency and elevation angle in Table 2. The constant term (36°K) is the ground-noise contribution from $\alpha_G = 0.124$ and $T_G = 290°K$. A factor of 10 has been included in the sun-noise term, so that the term represents a sun noise temperature 10 times the quiet level. The symbol θ is the elevation angle of the axis of the main beam of the antenna.

The antenna temperatures resulting from these assumed conditions are computed for the case of $\theta = 1$ degree at several frequencies (Table 3) as an illustration of the relative contributions of the various sources. In this calculation, the number of significant

*In a memorandum dated April 28, 1958.

†According to H.G. Weiss of MIT Lincoln Laboratory in a lecture on radar astronomy systems, at the Massachusetts Institute of Technology Special Summer Session on Radar Astronomy, August 23, 1960; printed Lecture Notes, p. 4.

figures retained in the result is of course greater than the precision of some of the data on which they are based. This is done to illustrate the relative significance of the various contributions to the total antenna noise temperature.

Table 4 gives the antenna-temperature results of similar calculations for several elevation angles, rounded off to a maximum of three significant figures. These results are plotted as Fig. 1.

If it is desired to modify the value given by the curve to correct for conditions that differ from those assumed here, the necessary modification will usually consist simply of addition or subtraction of an amount that may be computed by reference to Eq. (14). That is, the difference in the term of Eq. (14) that refers to the different assumed conditions may be computed, and this difference added or subtracted from the value given by the curve. It is especially likely that the ground-temperature term, here assumed to have a fixed value of 36°K, would be subject to modification insspecial cases. In computing the values for the terms of Eq. (14), it may of course be necessary to refer to Eqs. (10) to (13) and the related material of the text.

Table 3
Antenna Temperature Computation
for 1-Degree Elevation Angle

Freq. (Mc)	$1/L_T$	$T_{a(C)}$ (cosmos)	$T_{a(S)}$ (sun)	$T_{a(T)}$ (troposphere)	$T_{a(G)}$ (ground)	T_a
100	0.98	2420	47	5	36	2508
200	0.95	587	41	14	36	678
300	0.91	250	30	22	36	338
600	0.86	58	19	36	36	149
1000	0.82	21	14	47	36	118
3000	0.79	2	2	54	36	94
10,000	0.74	0.2	0.4	68	36	105

Table 4
Antenna Temperature (°K) as a Function
of Elevation Angle and Freqency

Freq. (Mc)	$\theta = 0°$	$\theta = 1°$	$\theta = 2°$	$\theta = 5°$	$\theta = 10°$	$\theta = 90°$
100	2480	2510	2510	2530	2540	2550
200	674	678	692	695	698	702
300	342	338	344	347	347	347
600	163	149	145	137	132	130
1000	136	118	103	90	83	77
3000	125	94	80	61	52	45
10,000	144	105	86	62	50	40

PART II - SYSTEM NOISE TEMPERATURE

BASIC CIRCUIT AND NOISE-TEMPERATURE PRINCIPLES

The curve for antenna noise temperature developed in Part I provides one element needed for calculation of the total noise power in a receiving system. In this part, the other elements in the calculation will be discussed.

As previously mentioned, this report is one of a series written in support of a procedure for radar range calculation (1 - 4), although as indicated in the introduction it has other applications as well. Therefore, some care will be taken to present a complete structure of reasoning, including some rather elementary matters, to avoid possible misunderstanding or vagueness.

Components of a Receiving System

The basic elements of a receiving system are shown in Fig. 3, which is a generalization of Fig. 2, Part I. It comprises a source (antenna), a load (receiver), and an interposed four-terminal network (transmission-line system, including such components as a duplexer, rotating joints, couplers, etc.). The four-terminal network is more generally designated a twoport transducer, so as to include the possibility of waveguide elements in which there are no identifiable terminals. The discussion here will sometimes be expressed in terms of a four-terminal network for convenience, but the ideas and results are equally applicable to the more general twoport transducer.

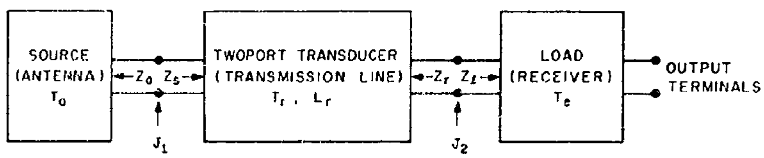

Fig. 3 - Schematic representation of a basic receiving system

The preliminary discussion will be in terms of a single-response receiver. Equation (2) and the definition of receiver input temperature by Eq. (4) are not applicable to calculating the noise temperature of multiple-response systems.

A still more general representation of the receiving system would show more than one four-terminal network interposed, in cascade, between the source and the load, and this case will be considered, as will also the multiple-response case. However, the basic principles of the system noise temperature calculation may be developed in terms of Fig. 3.

The source (antenna) is characterized by an effective (antenna) noise temperature, T_a, given by Fig. 1 of Part I for typical environmental conditions and a typical directive antenna, and by an internal impedance Z_a (in some cases, or generally, equal to the radiation resistance of the antenna). The four-terminal network (receiving transmission line) is characterized by an input (sending-end) impedance Z_s, an output (receiving end) impedance Z_r, a power loss factor L_r, a thermodynamic (thermal) temperature T_t, and an effective noise temperature T_r. The load (receiver) is characterized by an input impedance Z_ℓ, and an effective noise temperature T_e which is related to its noise factor $\overline{NF}$. In Part I the equations relating the system noise temperature, T_N, to these component characteristics were given for a single-response system:

$$T_N = T_a/L_r + T_r + T_e \tag{2}$$

$$T_r = T_t\ (1-1/L_r) \tag{3}$$

$$T_e = (\overline{NF} - 1)\ T_0 \tag{4}$$

where T_0 is 290°K. The reasoning on which these equations are based, and more precise definitions of the various quantities involved, will now be given. It is necessary first to consider some principles of circuit theory which have special significance in these matters.

In all of the ensuing discussion, certain assumptions are made concerning the circuits and the noise. Linearity of the circuits is assumed. This condition is usually realized in practical receiver circuitry for voltages of the order of the noise level. The noise itself is assumed to be of the thermal type, i.e., of uniform spectral distribution ("white" noise) and Gaussian amplitude distribution, although the latter condition is not essential. The noise voltages from separate individual sources within the receiving system are assumed to be statistically independent, i.e., uncorrelated, noncoherent. This condition is also usually met in practical receivers. Because of this assumption, voltages from separate noise sources combine at any common point in such a way that the resultant mean-square voltage is the sum of the mean squares of the individual voltages at the point, which means that the noise powers are directly additive.

Available Power, Gain, and Loss

It is to be noted that the foregoing equations do not explicitly contain the input and output impedances of the system components, nor ratios of these impedances, although it is well known that the transfer of power from one component to another decidedly depends on the impedance ratios. This desirable simplification of the expressions results from definitions of certain quantities which take into account the effects of impedance match or mismatch at the junctions J_1 and J_2 in Fig. 3. Specifically, the possibility of mismatch of impedances at J_1 is taken into account in the definitions of T_a and L_r, and mismatch at J_2 by the definitions of T_r, $\overline{NF}$, and T_e. These definitions are based on the concept of available power, and the corollary ideas of available gain and available loss.

The concepts of available power and available gain in calculation of the signal-to-noise ratio were discussed by Friis (23) in 1944, in his classic paper setting forth a rigorous standard basis for defining receiver noise factor. (Incidentally, Friis used and preferred the term "noise figure" in his paper, but D. O. North, in his also-classic paper (24) which preceded Friis' in 1942, used "noise factor," and this usage has been given preference by the IRE (25) although many engineers still follow Friis in the use of the term noise figure. It is now possible that this minor controversy will be settled by supervention of the term receiver noise temperature. It is evident that noise factor and noise temperature are alternative ways of expressing the same property of a receiver, or in fact of any component of a receiving system. The noise temperature terminology is becoming increasingly used and accepted, as the many references listed at the end of this report attest.)

The need for taking into account the possibility of impedance mismatch is primarily based on the fact, as shown by Llewellyn (26), that mismatch at J_2, Fig. 3, may actually be desirable, resulting in improved signal-to-noise ratio. If this were not so, it would be reasonable to assume matched impedances everywhere, but evidently this assumption cannot be made. On the other hand, it would be inconvenient, to say the least, if the expressions for system noise power explicitly contained the impedance ratios, which are seldom measured and known. The available power, gain, and loss concepts make it unnecessary to know them directly.

The available power at the output terminals of a power source is that which would be delivered to a load whose impedance is a conjugate match to the impedance of the source. (A simple analysis shows that a source delivers the maximum power of which it is capable to a load of this impedance, hence the term "available" power.) Available gain of a network is the ratio of the available power at its output terminals to the available power at its input terminals (i.e., from the driving source). Available loss is the reciprocal of available gain, and the term is customarily used in lieu of less-than-unity gain.

As Friis noted, although the gain or loss of a network as thus defined does not depend on the impedance match or mismatch at its output terminals, it does depend on the impedance relationship at its input terminals — in fact, it takes it fully into account. Therefore, in Fig. 3, any mismatch at J_1 is accounted for if L_r is defined as the "available loss" factor, for purposes of calculating the effect of the network on power made available at J_2 from the source. As will be shown later, this definition of L_r is also necessary in computing the effective noise temperature, T_r, of the receiving transmission line, Eq. (2).

The receiver noise factor was defined by Friis in terms of the available signal-to-noise ratios at the receiver input and output, and the official IRE definition (25) is equivalent to Friis', although expressed differently. Therefore, this definition automatically takes into account the effect of any mismatch at J_2 in Fig. 3. It is assumed, however, that the receiver noise factor used in calculations is the one that would apply for the match or mismatch actually existing at J_2, or, more restrictively, that optimum mismatch obtains both in measurement of the receiver noise factor in the laboratory and in its field operation. These statements also apply to noise temperatures.

If the impedances at a junction, such as J_1 of Fig. 3, are mismatched, it is evident that less power will be delivered to the input terminals of the network than in the matched-impedance case, and that the available loss, L_r, will be greater. If the impedance of the source, Z_a, has a real component R_a, and if the input impedance of the network, Z_s, has a real component R_s, it may be shown that the ratio of the available power to that actually delivered to the network is

$$\frac{P_{available}}{P_{delivered}} = \frac{|(Z_a + Z_s)|^2}{4\, R_a\, R_s} \tag{15}$$

which has the value unity when Z_a and Z_s are complex conjugates of each other. This ratio may be denoted by the symbol L_1 and called mismatch loss factor. If the network is also characterized by a dissipative loss factor L_2, defined as the ratio of the actual input power to the available output power, it is evident that the available loss, L_r, is the product $L_1 L_2$.

It is thus evident that a knowledge of the dissipative loss alone is in principle insufficient for evaluating L_r in Eqs. (2) and (3). Fortunately, however, L_1 is seldom a loss of appreciable magnitude, and in any case its magnitude may be readily determined by using Eq. (15). At first glance, it might seem that determination of Z_s, at J_1 of Fig. 3, would be difficult, because of the previously mentioned mismatch of impedances that commonly exists at J_2, since the impedance Z_s is the result of transmission-line transformation of Z_ℓ. But, the value of Z_s to be used in determination of L_1 is that which would be observed when there is an impedance match at J_2, in accordance with the definition of L_r as the available loss. Therefore, although in actual operation of the system there may be an appreciable and unknown mismatch at J_2, this mismatch is of no concern in the calculation of L_1.

Ordinarily (though not necessarily) the transmission line will have a characteristic impedance which, either directly or through a transformer, matches the antenna impedance Z_a, and similar matching is ordinarily maintained at all points throughout the receiving system except at J_2. But if an impedance match is assumed at J_2, the result is a completely matched system. This means that ordinarily there is no mismatch loss at J_1 under the circumstances that should be assumed for computing L_r.

Thus in most practical cases the available loss is equal to the dissipative loss. It may even reasonably be asked, is it possible to have a condition of mismatch at J_1 when a match is assumed at J_2? If the four-terminal network (e.g., transmission line) were completely lossless, an impedance match at J_2 would indeed imply a match at J_1, in accordance with a well-known theorem of circuit theory. But, this is not a necessary consequence when the network is lossy. However, in order for a mismatch to exist at J_1 when the impedances are matched at J_2, it is necessary for the network to be lossy and for its characteristic impedance to be mismatched at J_1. Thus, although it is unlikely to occur in practical systems, it is possible to have a mismatch loss at J_1 applicable to the calculation of the available loss L_r.

Therefore, this aspect of the available-loss concept is theoretically required. The most important practical aspect of the concept, however, is that the dissipative loss factor, L_2, to be used is that which would result with a matched load at J_2. As is well known, a mismatch at J_2, with resultant standing waves on the transmission line, can result in greater actual dissipation in the line than would occur with a matched load. This increase can be calculated for a known degree of mismatch and standing-wave ratio. But again, this effect is of no concern in the calculation of available loss (since a matched load is assumed) except in the unlikely case where the impedance observed at J_2, looking back toward the antenna, is not equal to the characteristic impedance of the transmission line.

The idea of available power, and especially its application to problems of the type just discussed, is directly based on Thévenin's theorem. That is, the idea that there is an available power at J_2, based on an available loss calculated as just described, and that it is related to the power actually delivered to a mismatched load by a mismatch loss factor of the type given by Eq. (15), is justified by Thévenin's theorem. Although this proposition might seem sufficiently obvious that it hardly needs justification, it is somewhat remarkable when it is realized that it automatically takes into account such mismatched-load effects as the increased dissipation in the transmission line due to a standing wave.

Thévenin's theorem, in approximately the form given by Everitt (27),* states that whenever an external load impedance (e.g., Z_ℓ at J_2, Fig. 3) is connected to any two terminals of a network containing any arrangement of generators and linear impedances, the current that flows in Z_ℓ can be computed by regarding the two terminals as the terminals of a simple generator, whose internal generated voltage is that which is measured at the actual terminals before the connection of Z_ℓ, and whose internal impedance is the impedance of the network viewed from these same terminals, with all generators replaced by impedances equal to the internal impedances of these generators. The idea of an open-circuit voltage at a pair of terminals, in conjunction with the concept of an internal impedance in the simple-generator sense, is entirely equivalent to the concept of available power; if the open-circuit voltage is e_0 and the resistive (real) part of the impedance is R_0, the available power is simply $e_0{}^2/4R_0$.

*It has been pointed out to the author by Lee E. Davies of Stanford Research Institute that this statement of Thévenin's theorem is correct only if the network contains no dependent generators (e.g., cathode followers). Mr. Davies states that the correct statement for this more general case is given by Seshu and Balabanian in "Linear Network Analysis" (Wiley, 1959).

In all of the foregoing discussion, the impedance at a pair of terminals is meant in the sense of Thévenin's theorem, unless qualified otherwise; that is, it does not refer directly to the characteristic impedance of the transmission line, but rather to the impedance that would be measured at the terminals. Thus for example if the network in Fig. 3 is a simple transmission line (but not necessarily a lossless one) the impedance Z_r at J_2 is given by the well-known transmission-line equation

$$Z_r = \frac{Z_0\,(Z_a + Z_0 \tanh \gamma \ell}{Z_0 + Z_a \tanh \gamma \ell} \tag{16}$$

where Z_0 is the characteristic impedance of the transmission line, γ is the propagation constant, and ℓ is the length of the line. (As is well known, and evident by inspection of the equation, if $Z_a = Z_0$, then also $Z_r = Z_0 = Z_a$.) The sole nontrivial aspect of this example is that Eq. (16) is ordinarily regarded as expressing the impedance observed at the <u>input</u> terminals of a line, but it also expresses the output-terminal impedance in the Thévenin-theorem sense, with the source impedance, Z_a, in the position ordinarily considered to be the load impedance.

These elementary matters have been discussed, as previously implied, to avoid misunderstandings which frequently occur in discussions of this type.

The Referral Concept

The system noise temperature, T_N, in conjunction with Boltzmann's constant, k, and the system bandwidth, B, expresses the total noise power of a receiving system. The only point in the system at which this total noise actually exists is at the output of the receiver. However, this output noise power is usually expressed in terms of an equivalent noise power at some earlier point in the system, known as the reference point. By "equivalent" it is meant that if the actual noise sources, distributed along the receiving system at various points, were somehow eliminated without otherwise disturbing the impedances, gains, and losses of the system, and replaced by a single equivalent source at the reference point, the noise power output would be the same. The system noise temperature, T_N, expresses the available noise power of this equivalent source, which is $k\,T_N\,B$. If the net available power gain from the reference point to the receiver output is G, the actual noise power output of the receiver is equal to $G\,k\,T_N\,B$. Thus it is important to state what reference point has been chosen when the system noise-temperature concept is employed.*

In general, the effect of a noise source at one point in the system may be <u>referred</u> to another point by multiplying or dividing the source noise power by the net power gain factor of the components between the two points. The source power is multiplied by the gain if it precedes the reference point, and divided if it follows. The reference values thus obtained for each individual source are then added together to obtain the total noise power at the reference point. Since noise power and noise temperature are proportional to each other for constant bandwidth, referenced noise temperatures may be directly calculated and added in this same way. The assumption is made that all the individual noise sources are statistically independent; if they are not, considerations of relative phase must be taken into account in calculation of their combined effect at the reference point. Ordinarily this is unnecessary.

When there is a net loss rather than a gain between two points of a system, the reference value may be obtained by realizing that multiplying by gain factor is equivalent to dividing by loss factor, and vice versa, since the available loss is defined as the reciprocal of the available gain.

*After initial preparation of this report, an excellent discussion of this important point appeared as a letter to the editor of the Proceedings of the IRE, from Edwin Dyke, captioned, "Correction and Coordination of Some Papers on Noise Temperature" (Apr. 1961; vol. 49, p. 814).

A signal power may be referred to an arbitrary point of a system in the same manner. Ordinarily there is only one signal source, but if there should be more than one and if they are coherent (phase-related), the relative phases must be taken into account in obtaining the total effective signal power at the reference point, as in the case of correlated noise sources.

When both signal and noise powers are referred to the same point in a system in this way, their ratio is the same as the output signal-to-noise ratio. Also, although the power values thus calculated are available powers rather than actual powers, the ratio is the same as the ratio of the actual powers. It is for this reason that the available-power concept is so useful.

The choice of a system reference point is completely arbitrary in principle, but the customary choice is the input terminals of the receiver. This is the reference point that was assumed in deriving Eq. (2), as will be shown.

The referral principle also applies in the assignment of noise temperatures to individual components of a system. The reference point may be either the input or the output terminals (port).

Bandwidth and Gain Definitions

Since the bandpass characteristic of a receiver is never truly rectangular, its width becomes a matter of arbitrary definition. A common practice is to define it as the frequency separation of the half-power points on the response curve. In noise power calculation, however, it is necessary to employ the noise bandwidth, defined by the following expression, as given by North (24) and others:

$$B_n = \frac{1}{G_0} \int G(f)\, df \tag{17}$$

where $G(f)$ is the gain-frequency characteristic and G_0 is its value at the nominal frequency of the passband. The integral is taken over a frequency interval that encompasses all significant response within the passband whose noise bandwidth is being evaluated. When B_n is evaluated at the system input terminals, with $G(f)$ the overall system gain, the resulting quantity is called the system noise bandwidth, B_N, with the further provisions that for multiple-response systems the integral of Eq. (17) is taken over the principal-response passband only and G_0 is then the nominal frequency of the principal response, denoted $G(f_p)$.

Fortunately, as shown by Lawson and Uhlenbeck (9, p. 177), the half-power bandwidth and the noise bandwidth are not greatly different in typical cases.* The noise bandwidth is generally somewhat greater. The difference is appreciable in the case of a single singly-tuned amplifier stage, being about equivalent to 2 decibels in the noise power that would be calculated from the expression KTB. But as the number of cascaded tuned stages is increased the difference decreases, being equivalent to only 0.6 decibel for four stages. A further decrease occurs if the stages are doubly or multiply tuned and properly coupled, being only 0.2 decibel for two doubly-tuned stages. Therefore, if a highly precise evaluation of the receiver noise power is required, the noise bandwidth rather than the half-power bandwidth must be used, but for practical purposes the latter quantity is adequate in most cases.

*The analysis of this matter is credited by Lawson and Uhlenbeck to A. M. Stone of the MIT Radiation Laboratory, RL Report 708, June 22, 1945.

The system noise temperature, T_N , is actually a way of expressing the system noise power output, as the product $k\,T_N\,B_N\,G$, where G is the available gain from the system reference point to the system output, and B_N is the system noise bandwidth. It is apparent that some arbitrariness is permissible in the definitions of B and G, since however they are defined, T_N may then be assigned the temperature value that results in the correct value of the product $k\,T_N\,B_N\,G$. The principal problem in defining G is that of defining the "nominal frequency" of the system, f_0, such that $G(f_0)$ becomes the nominal gain, G_0, in Eq. (17). The selection of the nominal frequency is essentially arbitrary. The value chosen will ordinarily be at or near the center of the response band, and this choice is assumed in some of the discussions that follow,* as well as in the preceding discussion of the relative values of the noise bandwidth and half-power bandwidth. But any value chosen for f_0 will result in the same value of the product $B_N\,G_0$, as Eq. (17) indicates, and this product is the quantity of real physical significance. However, once a selection of f_0 has been made, all calculations of power involving noise temperatures and the quantity B_N must be made by using the gain $G(f_0)$.

As Eq. (17) indicates, the significant physical quantity is the gain-frequency function, $G(f)$, and its integral, but calculations based on the use of B_N and G_0 are much simpler to make. These quantities are directly applicable to the calculation of noise power output when the input noise spectrum is uniform (white noise) and the signal is of a bandwidth that permits its passage through the system without appreciable waveform distortion. In this case, if the input noise power density is S_N (independent of frequency), the output power is $S_N\,B_N\,G_0$. Also, an effective input signal power, P_s, can be defined such that the signal output power is $P_s\,G_0$. In the case of a cw signal at the frequency f_0, the validity of this calculation is obvious. When the signal is characterized by a spectrum of frequencies rather than a single frequency, the effective signal power input depends on the shape and width of this spectrum in relation to the function $G(f)$. Similarly, when the noise spectrum is not uniform over the receiver passband, an effective or equivalent uniform-spectrum value may be calculated and represented by a noise temperature, such that the product $k\,\bar{T}\,B_N\,G(f_0)$ represents the actual noise output power. (The notation $\bar{T}$ is used to indicate that noise temperature used in this way implies a weighted average of the values applicable to differential frequency intervals over the receiver passband.)

Multiple-Response Systems

Complications arise in the definition of such quantities as gain, bandwidth, and noise temperature when the system incorporates a multiple-response transducer. Part of the problem is to state the definitions unambiguously without using excessively complicated notation which may create confusion because of its complexity. Therefore, some degree of insight on the part of the reader is usually assumed, and unfortunately may be required in what follows. An aid to acquisition of such insight is the following excellent definition of a multiple-response transducer, which is quoted from Ref. 28:

"Multiple-response receivers are those in which more than one frequency applied to the accessible input terminals corresponds (by way of transformations) to a single output frequency and vice versa. We denote the multiplicity of the responses by counting the number of frequencies which, if applied to the accessible input terminals of the system, contribute significantly to a single output frequency within the desired output band. ... By single-response receiver is meant any receiver in which only one frequency at the accessible input terminals corresponds to a single output frequency, regardless of the complexity of the gain-frequency characteristic."

*The discussions referred to are those in which it will be indicated that the average noise temperature is virtually the same as the spot temperature at the nominal response frequency, for certain conditions on the functional behavior of the spot temperature, as will be demonstrated mathematically in Appendix A.

In defining noise bandwidth by Eq. (17) for a multiple-response system, it is important to state what limits of integration are intended, and to which of the response passbands G_0 refers. As previously stated, the system noise bandwidth, B_N, is defined at the system input terminals by limiting the interval of integration to the principal-response passband, as will subsequently be indicated by the symbol $\int_p$, and by using for G_0 the overall system gain at the nominal frequency of the principal response, f_p , designated $G(f_p)$. However, it will at times be appropriate to refer to the "noise bandwidth over all responses," which will mean that the integration of Eq. (17) is to be carried out over all the response bands, but with $G_0 = G(f_p)$. In still other cases it may be necessary to define the noise bandwidth of the i^{th} response band, for which G_0 will be taken as $G(f_i)$, where f_i is the nominal frequency of the i^{th} response band (the frequency in the i^{th} band that is converted to the same output frequency as is f_p), and the integration of Eq. (17) is carried out over the i^{th} response band only. Related considerations will arise in defining the input noise tempe ature of a multiple-response transducer.

In this report, a multiple-response transducer will be considered to have more than one input response frequency but only one output frequency channel. Although heterodyne transducers actually have multiple output channels as well as multiple input responses, usually only one output channel is utilized, the others being suppressed by filtering. Therefore the restriction of the treatment here to the case of a single output response channel is believed not to be a serious practical limitation.

NOISE-TEMPERATURE DEFINITIONS

IRE Standard 57 IRE 7. S2 (Ref. 25) defines noise temperature at a port (pair of terminals) as "the temperature of a passive system having an available noise power per unit bandwidth equal to that of the actual port, at a specified frequency." It is further noted that any passive circuit (e.g., a resistor, by itself or in combination with passive reactance elements) at thermodynamic temperature T provides an available noise power density equal to kT watts per cycle-per-second of bandwidth, where k is Boltzmann's constant (1.38×10^{-23} watt-seconds per degree).

Spot-Frequency and Average Noise Temperature

Significant features of this basic definition are: (a) a noise-power spectral density, rather than a total power, is described; (b) this density is in general a point function of frequency — that is, in principle a different value of temperature may exist for every possible value of frequency. In the language of the IRE noise-factor standards (25), this conception of noise temperature would be designated "spot noise temperature."

When the noise temperature concept is actually used to describe a power, rather than a power density, a specific noise bandwidth is implied. In such cases the temperature is not regarded as a point function of frequency, but as a single value which, multiplied by Boltzmann's constant and the width of a particular passband, represents the total available power within the band, referred to a specified port. It may be thought of as an equivalent uniform temperature with which the actual frequency-dependent temperature could be replaced without changing the noise power output of the device or system. It is analogous to the average noise factor, and will here be called the average noise temperature. Although the terms spot and average have not been applied to noise temperature definition in the existing IRE Standards, and are not used (to the author's knowledge) in the existing literature, the two different temperature concepts are in common use. The antenna noise temperature, as plotted in Fig. 1 of Part I is a good example of a spot noise temperature, while the system noise temperature, T_N of Eq. (2), is an example of an average noise temperature. Possibly the reason that the distinction between these two types of noise temperature is not generally made explicitly is that the average temperature and the spot value near the center of a passband are usually equal or very nearly equal (as will later be shown in Appendix A). But, the distinction is sometimes necessary, especially in analysis of multiple-response systems where the spot temperature is typically a different value in each of the different responses, and in any case the distinction exists theoretically. The spot temperature is the more basic quantity, while the average temperature is an engineering convenience. Therefore the general procedure in the analyses that follow will be to present first the spot-frequency expressions, and then to give the equivalent average-temperature expressions.

Component and System Noise Temperatures

Noise temperature may describe the output noise power or power density of a component (e.g., antenna, signal generator, termination, active or passive transducer) or of a system of such components in cascade. In all cases it is the available output power or power density that is described, but this may be done in terms of an equivalent power or power density at any other port or pair of terminals, by using the referral principle.

The noise temperature of a system of cascaded components is expressible in terms of the individual component noise temperatures. Sometimes relatively simple formulas for the overall noise temperature of a system are given without specifying the limited conditions for which they apply and without specifying the ports or terminals to which the various temperatures are referred. These matters will be discussed in some detail. Generally, the term system noise temperature describes the total system power output; i.e., an average temperature is meant. This meaning will be implied in this report also, although in principle the system noise output power density may also be described in terms of a system spot noise temperature.

The concept of a system noise temperature is a natural extension of the receiver noise-temperature definition, and has been employed by numerous workers in one form or another (not always explicitly). In fact, the analogous extension of the receiver noise factor concept was made by D. O. North (24) in 1942 and by Norton and Omberg* in 1947. Norton† has subsequently refined this concept and gives an expression for an effective (system) noise factor (figure) that is directly analogous to the system noise temperature expressed by Eq. (43) of this report. (Norton's equation is limited to single-response systems, and assumes that the thermodynamic temperature of the antenna and transmission-line lossy elements is equal to the noise-factor reference temperature.) The basic ideas of North's "operating noise factor," Norton's "effective noise figure," and the "system noise temperature" are the same, namely, to express the total noise power of the receiving system, including sources external to the receiver, rather than merely the intrinsic noise of the receiver itself, or the intrinsic noise plus that of a standard-temperature input termination.

Noise Temperature of a Twoport Transducer

The noise temperature of a twoport transducer (e.g., receiver), as used in this report, agrees with that of IRE Standard 59 IRE 20. S1 in the case of a single-response system, but not otherwise. The definition used here expresses the noise power output of the transducer due solely to sources of noise within the transducer itself. It does not include any noise-power contribution from the input termination, as does the IRE-defined quantity T_e in the case of a multiple-response transducer. (This statement is based on the author's interpretation, as will be explained in Appendix B, of the IRE definition, in which a number of qualified persons concur, although other interpretations are possible.) A redefinition

*K. A. Norton and A. C. Omberg, "The Maximum Range of a Radar Set," Proc. IRE 35:9-14 (Jan. 1947).

†K. A. Norton, "Transmission Loss in Radio Propagation," Proc. IRE 41:150-151 (Jan. 1953).

is believed to be necessary for system noise-temperature calculation, or at any rate it avoids unnecessary complications in the case of multiple-response systems.

IRE Standard 59 IRE 20. S1 (Ref. 25) defines the "effective input noise-temperature of a twoport transducer (T_e)" and relates it to the spot noise factor by Eq. (4). There are no difficulties with this definition when it is applied only to single-response systems, to which Eq. (2) is restricted. Because of the difficulty that arises in the case of a multiple-response system, the starting point for noise-temperature definition in this report is the more basic one of Standard 57 IRE 7. S2, which has been quoted. This is a spot noise temperature.

Transducer output noise temperature is here defined by assuming that the input termination is noise-free (e.g., a passive network at zero thermodynamic temperature). Under these conditions a certain available noise power density, $S_N(f)$ (watts per cycle-per-second), will result at the transducer output terminals, and obviously this noise power is due solely to sources within the transducer itself. The output noise temperature is then defined as the Kelvin temperature, $T_{(o)}(f)$, such that $k\,T_{(o)}(f)$ is equal to $S_N(f)$.

The effective input temperature, $T_{(i)}(f)$, is defined as this output temperature divided by the available gain of the transducer, $G(f)$. In the case of a multiple-response heterodyne transducer, $G(f)$ may be taken as the conversion gain of any one of the input responses for the purpose of defining the input temperature, which may then be said to be referred to that response. The response so chosen is usually designated the principal response, the others then being considered spurious responses if the principal response is the one in which signal is received. More generally, or when signal is received or receivable in other responses as well, the response thus chosen may simply be designated the reference response. However, the term principal response will be used here, and the input noise temperature thus defined will be called the principal-response input noise temperature, T_p. In the case of a single-response transducer its definition coincides with that of the IRE-defined input noise temperature, T_e.

In applying this definition to multiple-response transducers it is necessary to remember that this input temperature is basically the output temperature referred to a particular input response, and that by itself it completely describes the total transducer noise power density. In other words, when multiplied by Boltzmann's constant and the conversion gain at the reference response, it gives the total output noise power density. It might otherwise be thought that the noise temperatures assignable to each of the input responses should be thus multiplied and the results added together to obtain the total noise power. In fact, as will be shown later, a procedure of this nature is applicable to a broad-band single-response transducer that precedes a multiple-response transducer; but for the reasons just stated, it is not applicable to the multiple-response transducer itself.

In certain equations that follow (19a, 22a, 25 and 30 in particular), definite integrals with the frequency limits zero to infinity contain a transducer input noise temperature in the integrands. When the transducer is a multiple-input-response heterodyne converter, a consideration similar to the one just described applies; i.e., this temperature is not to be taken as all of the separate input-response temperatures, but only as one of them, usually the principal-response temperature. Generally this rule is properly taken into account by performing the integration over the frequency interval of the principal response only. The rule is applicable, however, only to multiple-response transducers, as previously defined.

Effect of Input Termination

Although the input termination makes no contribution to the noise power expressed by this transducer noise temperature, it does affect both the input and output temperature values (generally), because noise current generated within the transducer may flow in the input termination. Therefore (as may be intuitively obvious and can also be shown rigorously) the impedance of the input termination affects the transducer noise power output. Consequently, the noise temperature rating of the transducer, as here defined, depends upon the impedance, but not upon the temperature, of the input termination.

Apart from this consideration, the impedance of the input termination affects the input temperature because $G(f)$, the transducer gain, by which the output temperature is divided to obtain the input temperature, is the available gain. This quantity includes the effect of impedance mismatch (if there is any) at the input terminals, as has been discussed. The transducer input noise temperature expresses an equivalent available input power, and this will be a function of the input-termination impedance even in those cases in which this termination does not affect the transducer output noise power.

For these reasons, a statement of transducer noise temperature is incomplete unless the impedance of the input termination is also stated. If it is not explicitly stated, however, it may be tacitly understood that the temperature rating is the one that applies with the input termination actually used, or with an optimum input termination (the one that results in the lowest possible input noise temperature), according to the context. These considerations are of course the same as those stated (25) in connection with transducer noise-factor ratings.

Spot-Noise-Temperature Analysis of a Cascade System

In order to develop the definitions of average component and system noise temperature it is necessary to show how the total noise output of a system is expressed in terms of the component spot noise temperatures.

Figure 4 depicts a cascade of n components, of which the first is a one-port source (e.g., antenna or signal generator) and the remainder are twoport transducers of any type (e.g., passive networks, single-response amplifiers, heterodyne converters). The j^{th} component is characterized by an output spot temperature, $T_{(o)j}(f)$, and an available power gain $G_j(f)$. The portion of the system comprising the components that follow the j^{th} component has an overall gain given by

$$\mathcal{G}_j(f) = \prod_{k=j+1}^{n} G_k(f) \tag{18}$$

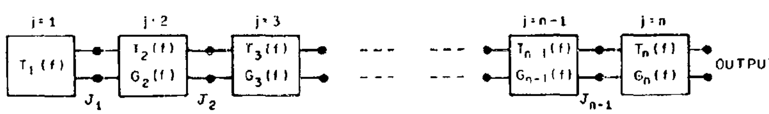

Fig. 4 - Receiving system with n cascaded components

In terms of these quantities, the total available noise power output of the system is

$$P_N = k \sum_{j=1}^{n} \left[\int_0^{\infty} T_{(o)j}(f)\, \mathcal{G}_j(f)\, df \right] \tag{19}$$

Since the input noise temperature of the j^{th} component is by definition

$$T_{(i)j}(f) = T_{(o)j}(f)/G_j(f) \tag{20}$$

the total noise power expressed in terms of component input temperature is

$$P_N = k \int_0^\infty T_{(o)1}(f)\, \mathcal{G}_1(f)\, df + k \sum_{j=2}^{n} \left[\int_0^\infty T_{(i)j}(f)\, \mathcal{G}_{j-1}(f)\, df \right] \tag{19a}$$

Average Noise Temperature in Cascade Systems

The quantity P_N is the basis of definition of the system noise temperature, T_N, which is an average noise temperature since it describes the total power within the system noise bandwidth, B_N, as defined following Eq. (17). The system noise temperature at the system output terminals is simply

$$T_{NO} = P_N/(k\ B_N)\ . \tag{21}$$

The value referred to the junction J_R, at the output of the R^{th} component, is

$$T_{NR} = P_N/(k\ B_N\ \mathcal{G}_R) = \frac{P_N}{k\ B_N} \prod_{k=R+1}^{n} (1/G_k)\ . \tag{21a}$$

(In subsequent formulas, the second subscript of T_{NR} will be omitted when the reference point is the input terminals of the receiver. The symbol T_{NI} will be used to denote the system noise temperature referred to the system input terminals, corresponding to the antenna or signal-generator output terminals — i.e., $R = 1$.)

The average noise temperature of a component in a cascade system may be similarly defined in terms of the contribution of the component to the total output power of the system, P_j — i.e., its contribution to P_N, which for the j^{th} component is the j^{th} term of Eq. (19) or (19a):

$$P_j = k \int_0^\infty T_{(o)j}(f)\ \mathcal{G}_j(f)\ df \tag{22}$$

$$P_j = k \int_0^\infty T_{(i)j}(f)\ \mathcal{G}_{j-1}(f)\ df\ . \tag{22a}$$

The average output noise temperature of the j^{th} component is defined to be

$$\overline{T}_{(o)j} = P_j/\left[k\ B_N\ \mathcal{G}_j(f_p)\right] \tag{23}$$

and the average input temperature is defined as

$$\overline{T}_{(i)j} = P_j/\left[k\ B_N\ \mathcal{G}_{j-1}(f_p)\right] \tag{23a}$$

where B_N is the overall system noise bandwidth, as defined following Eq. (17), and f_p is the nominal frequency of the principal response of the system. (For a single-response system the notation f_o may be used.)

It is important to note that the bandwidth in these component average-temperature definitions is the system noise bandwidth, which will generally be different from the component's own bandwidth. This is necessary in order that the referred component temperature contributions to the total system temperature shall be proportional to the referred power contributions.

From Eqs. (22), (22a), (23), and (23a)

$$\bar{T}_{(o)j} = \frac{\int_0^\infty T_{(o)j}(f)\, G_j(f)\, df}{B_N\, G_j(f_p)} \tag{24}$$

$$\bar{T}_{(i)j} = \frac{\int_0^\infty T_{(i)j}(f)\, G_{j-1}(f)\, df}{B_N\, G_{j-1}(f_p)}. \tag{25}$$

These expressions may be written in a way that emphasizes the analogy of these spot and average noise temperatures to the spot and average noise factors. If the system bandwidth is determined by the circuitry of the last cascaded component (as will ordinarily, or at any rate often, be the case), the denominators of Eqs. (24) and (25) may be written as integrals. This will be done, as an illustration, for Eq. (24):

$$\bar{T}_{(o)j} = \frac{\int_0^\infty T_{(o)j}(f)\, G_j(f)\, df}{\int_p G_j(f)\, df}. \tag{24a}$$

This equation is closely analogous to the equation given in IRE Standard 59 IRE 20. S1 (Ref. 25) for the relation between average and spot noise factors. The notation for the integral of the denominator indicates that it is to be taken only over the principal-response passband if the system has multiple responses.

Also in analogy with the noise-factor case, it is often true that the average noise temperature is equal or nearly equal to the spot noise temperature at the center of the passband. It is exactly equal if the spot temperature is constant over the band, or if the variation is exactly linear with frequency and the response is perfectly symmetrical about the center frequency. Appendix A presents proof of these statements, based on Eq. (24), for the single-response case.

If the j^{th} component precedes a multiple-response transducer having s responses, it may be shown in a similar way that Eq. (24) can be approximated as follows:

$$\bar{T}_{(o)j} = \frac{\sum_{i=1}^{s} T_{(o)j}(f_i) \int_i G_j(f)\, df}{B_N\, G_j(f_p)} \tag{26}$$

where the integral notation denotes integration over the i^{th}-response passband. This integral may also be written

$$\int_i G_j(f)\, df = B_{Nji}\, G_j(f_i) \tag{27}$$

where B_{Nji} may be thought of as the noise bandwidth of the i^{th} response as observed at the output of the j^{th} component. In these terms Eq. (26) becomes

$$\bar{T}_{(o)j} = \sum_{i=1}^{s} T_{(o)j}(f_i) \left[G_j(f_i)/G_j(f_p)\right] b_{ji} \tag{28}$$

in which $b_{ji} = B_{Nji}/B_N$, a quantity that is ordinarily equal to one. $T_{(o)j}(f_i)$ is the spot-frequency output temperature at the frequency of the i^{th} response.

This is a result of which considerable use will be made later. It can also be expressed in terms of input temperatures, in a form that is analogous to Eq. (25):

$$\overline{T}_{(i)j} = \sum_{i=1}^{s} T_{(i)j}(f_i) \left[\frac{G_{j-1}(f_i)}{G_{j-1}(f_p)}\right] b_{(j-1)i} \,. \tag{29}$$

(The dual use of the subscript i here may be confusing unless it is realized that the parenthetic (i) denotes an input noise temperature, while the nonparenthetic i denotes the i^{th} response. Similarly, in the following equations the parenthetic (o) subscript denotes an output noise temperature, while the nonparenthetic subscript zero in f_0 denotes the nominal frequency of a single-response system.)

For the single-response case, Eqs. (28) and (29) obviously reduce to

$$\overline{T}_{(o)j} = T_{(o)j}(f_0)\, b_j \tag{28a}$$

and

$$\overline{T}_{(i)j} = T_{(i)j}(f_0)\, b_{j-1} \,. \tag{29a}$$

When Eq. (29) is applied to the case of a multiple-response transducer, as opposed to a broad-band single-response component that precedes the heterodyne stage of the system, the equation also reduces to the form of Eq. (29a), with f_0 replaced by f_p, because $T_{(i)j}(f_i)$ exists only for the value of f_i corresponding to the principal-response frequency, f_p, as previously discussed.

Average Noise Temperature of an Isolated Transducer

The foregoing component average-temperature definitions are system-oriented, being expressed in terms of the system noise bandwidth and the gain-frequency function of the ensuing portion of the system. This form of definition is required in system noise-temperature calculation.

It is also possible, however, to define a transducer average noise temperature, $\overline{T}_c$, that is referred to the component itself, if the transducer has a well-defined bandwidth. The principal usefulness of such a definition would be as an index of the low-noise merit of the transducer, without reference to its use in a particular system.

For system noise-temperature calculation, both input and output component noise temperatures were defined, and both are useful under varying circumstances as will be apparent later when cascade-system formulas are developed. For transducer low-noise-merit rating by means of an average noise temperature, however, only the input value has significance. This is defined in terms of the input spot noise temperature, $T_{(i)c}(f)$ (which in turn is equal to the output spot temperature divided by the transducer gain, $G(f)$) as follows:

$$\overline{T}_{(i)c} = \frac{\int_0^\infty T_{(i)c}(f)\, G_c(f)\, df}{\int_p G_c(f)\, df} = \frac{\int_0^\infty T_{(i)c}(f)\, G_c(f)\, df}{B_{Nc}\, G_c(f_p)} \tag{30}$$

where the integral of both numerator and denominator are taken over the principal-response passband in the case of a multiple-response heterodyne transducer. B_{Nc} is the principal-response noise bandwidth of the component, and $G_c(f_p)$ is the available gain at the nominal frequency of the principal response.

This component-bandwidth-referenced temperature will often have the same value as the system-bandwidth-referenced value, Eq. (25), but not always. The most important case in which the values will be quite different is that in which the transducer has a broad passband which encompasses multiple responses of the following portion of the system described by $\mathcal{G}(f)$ in Eq. (25). The system-bandwidth-referenced average temperature will then be described by a summation as in Eq. (29), and will be considerably larger, numerically, than the component-bandwidth-referenced value of Eq. (30). However, as previously implied, it is probably not appropriate to apply the concept of component-bandwidth-referenced average noise temperature to a transducer that has a broad poorly-defined passband. Such components are meaningfully characterized only by their spot-frequency noise temperatures, when they are not associated with a particular system.

Comparison of Transducer Input-Noise-Temperature Definitions

A modification of the IRE-defined transducer input noise temperature, T_e, has been proposed here and denoted T_p, the principal-response input noise temperature. It differs from T_e only when the transducer is of the heterodyne type having multiple input responses.

A third definition has been proposed recently (28) by a panel reporting on a study of the noise-temperature method of specifying and analyzing system noise performance at the 1961 IRE PGMTT National Symposium (Washington, D.C., May 17, 1961). The quantity defined is termed the "broad-band" effective input noise temperature, T_b. It also differs from T_e, and from T_p, only in the multiple-response case.

T_e and T_p have the common property of being defined in terms of the principal-response passband, and of being assignable in principle to any one of the input responses. They may be considered as either spot-frequency or average temperatures. T_p and T_b have the common property of being «intrinsic" noise temperatures that do not include any contribution from a standard-temperature input termination. The quantity T_b, however, is purely an average temperature. It is defined in terms of the output power, P_n, that would result if the transducer were operated with a noise-free (zero temperature) input termination. If the actual transducer were replaced by a noise-free but otherwise equivalent transducer, T_b is the temperature of an input termination, common to all input responses, that would result in the same transducer output power, P_n.

The differences in the meanings of these three quantities are best illustrated by the expressions for transducer noise power output in terms of each of them. These expressions will be given for the transducer considered by itself, except for an input termination of appropriate impedance. The noise power expressed is in all cases that due solely to the transducer by itself — i.e., that which would result if the input termination were noise-free (at zero temperature). This will be done in terms of the quantity T_e as well as T_p and T_b even though the <u>definition</u> of T_e is in terms of a transducer with an input termination at the standard temperature $T_0 = 290°K$. To do this it is first necessary to define a quantity that may be described as the ratio of (a) the noise bandwidth of the transducer over all input responses to (b) its noise bandwidth in the principal response only:*

*This ratio is referred to in IRE Standard 59 IRE 20. S1 (Ref. 25) in connection with measuring the noise factor of a heterodyne transducer by the dispersed-signal-source method.

$$\beta = \frac{\int_0^\infty G_c(f)\, df}{\int_p G_c(f)\, df}. \tag{31}$$

To simplify matters, T_e and T_p will be considered to be spot temperatures that are constant over the principal-response passband, and hence equivalent to average temperatures. T_b, as previously mentioned, is by definition an average temperature. The noise-power-output expressions are then

$$P_c = k\left[T_e - (\beta - 1)\, T_o\right] \int_p G_c(f)\, df \tag{32a}$$

$$P_c = k\, T_p \int_p G_c(f)\, df \tag{32b}$$

$$P_c = k\, T_b \int_0^\infty G_c(f)\, df\,. \tag{32c}$$

The interpretation of the definition of T_e on which Eq. (32a) is based is explained in Appendix B. From these equations it is apparent that

$$T_e = T_p + (\beta - 1)\, T_o \tag{33a}$$

$$T_e = \beta T_b + (\beta - 1)\, T_o \tag{33b}$$

$$T_p = \beta T_b \tag{33c}$$

in which the term $(\beta - 1)\, T_o$ represents the contribution to the IRE-defined input noise temperature of a standard-temperature input termination in conjunction with the spurious responses. Since for a single-response transducer $\beta = 1$, it is apparent that for this case T_e, T_p, and T_b all have the same value.

It has been stated previously that T_e is not a convenient quantity for system-noise-temperature calculation (in the multiple-response case). The basis of this assertion is the presence in Eq. (32a) of the $(\beta - 1)\, T_o$ term, which does not describe any actual noise contribution but must be included solely because of the definition of T_e, and is therefore simply an unnecessary complication.

Inclusion of the $(\beta - 1)\, T_o$ quantity in the definition of T_e arises from its relationship to the IRE-defined noise factor, in which it was presumably considered desirable to take into account the noise-performance-degradation effect (in ordinary applications) of the so-called spurious responses. This was done in terms of a standard-temperature input termination. But as others have pointed out, this input-termination temperature is not a good representation of many present day receiving systems, and therefore T_e is not always a good index of multiple-response-receiver noise performance. Because of the great variability of antenna temperature, as indicated by Fig. 1, it seems equally valid to rate the noise performance of a multiple-response transducer by an intrinsic temperature, such as T_p or T_b, and a simplification of system-noise-temperature calculation thereby results.

As indicated by Eqs. (32b) and (32c), the noise power output of a transducer may be equally well represented in terms of either T_p or T_b. For system noise power computation, either may be used. Measurement of T_p probably requires a measurement of the quantity β, while T_b may be more directly measurable, although this depends on the particular technique of measurement employed. However, β or its equivalent must be measured to employ T_b in a system power calculation.

As indexes of the low-noise performance of receivers, T_p and T_b each have certain advantages. These are illustrated by the following example.

Consider two receivers, one having a single response and the other having two equal-gain responses. That is, the gains of the dual-response receiver in each response are equal to each other and to the gain of the single-response receiver. When connected to noise-free input terminations, let the total noise power outputs of the two receivers be the same. Since their gains are the same, both receivers have the same T_p rating. However, the value of β for the dual-response receiver is 2, so the T_b rating of the dual-response receiver is only half that of the single-response receiver.

An index of receiver noise performance should be monotonically related to the output signal-to-noise ratio for a constant value of input signal. The equal T_p ratings of these two hypothetical receivers correctly describe their <u>relative</u> performance when the input signal power is the same for each, and the input-termination noise temperature is zero. The T_b ratings would erroneously indicate that the performance of the dual-response receiver is better under these same conditions. With a nonzero temperature of the input termination, the overall noise performance of the dual-response system is poorer, a fact that the definition of T_e takes into account, so that from this particular point of view it has some virtue.

On the other hand, when the signal power <u>spectral</u> density is uniform over a frequency range encompassing both responses of the dual-response receiver, as in some radiometry applications, the T_b ratings are a significant measure of the relative performance, assuming that the total noise entering the receiver from the antenna is in this case regarded as "the signal," and that transmission-line noise is negligible. However, this statement is based on a different definition of what is meant by "a constant value of input signal." In terms of a constant value of signal <u>power</u> input, rather than uniform signal spectrum density, the T_p ratings are applicable.

As the foregoing discussion indicates, it is difficult to rate the merit of a transducer without considering its use in a system. The definitions of T_e, T_p, and T_b are each appropriate in different types of system applications. The only fully meaningful rating, however, is the <u>system input</u> noise temperature, T_{NI}, defined by Eq. (21a) with $R = 1$, for which formulas will be given in the following pages.

EQUATIONS FOR NOISE-TEMPERATURE EVALUATION

The system noise temperature is expressed by Eq. (2) and subsequent more general equations as the sum of contributions from the antenna, the transmission-line system, and the receiver. Evaluation of the antenna noise temperature has been discussed in Part I. In the following sections, evaluation of transmission-line noise temperature and the receiver noise temperature will be considered, and equations for calculating the noise temperature of a cascade system will be developed.

Transmission-Line Noise Temperature

The derivation of Eq. (3), expressing the noise temperature of a linear passive four-terminal network (e.g., transmission line or waveguide), has been given by Dicke (21) for the case of matched load impedance. Here Dicke's argument will be employed without the matched-load restriction. The derivation will be given in terms of Fig. 5, which is essentially a repetition of Figs. 2 and 3 except that the components are now specifically assumed to be linear passive elements, at specified thermodynamic temperatures which result in generation of noise power by each component in accordance with Nyquist's theorem.*

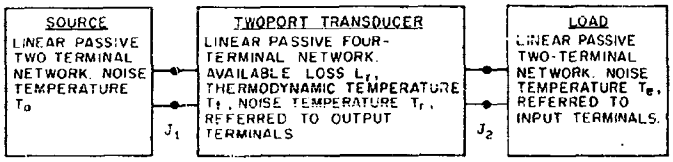

Fig. 5 - Arrangement of linear passive networks analogous to components of a basic receiving system

If the thermodynamic temperature of the source in Fig. 5 is T_a, then its noise temperature is also T_a, since it is demonstrable from Nyquist's theorem that the noise power density available from any linear passive source at temperature T_a is kT_a watts per cycle-per-second.

The networks to the right of J_1 may initially be assumed to be at zero thermodynamic temperature, so that they generate no noise power. In this case the available noise power at J_2 is $k\ T_a\ B/L_r$ where B is the bandwidth and L_r is the available loss of the four-terminal network. As previously noted, L_r is in principle the product of the dissipative loss and any mismatch loss that may exist at J_1, as given by Eq. (15).

Now suppose that the thermodynamic temperature of the four-terminal network is made equal to T_a all other temperatures remaining as before. The entire system to the left of point J_2 may now be regarded as a single linear passive network at temperature T_a, and hence the available noise power at J_2 must be $k\ T_a\ B$, by Nyquist's theorem. Since the contribution of the source at the left of J_1 to the available power at J_2 must still be the same as before (on account of the assumption of linearity of the system), the contribution of the four-terminal network, P_r, is the difference

$$P_r = k\ T_r\ B = k\ T_a\ B - k\ T_a\ B/L_r \tag{34}$$

from which it is deducible that

$$T_r = T_a\ (1 - 1/L_r)\ . \tag{34a}$$

*The reader is assumed to be acquainted with this most basic proposition, which states that if a resistance R is at thermodynamic temperature T, a root-mean-square noise voltage equal to $\sqrt{4kTRB}$ will appear across its open terminals, where k is Boltzmann's constant and B is the bandwidth.

Because of the linearity of the system, it is apparent that the noise power contributions of each of the two portions of the system must be independent of the thermodynamic temperature of the other. Therefore, if the four-terminal network is at any thermodynamic temperature T_t, its noise temperature T_r will be given by Eq. (34a) with T_t substituted in place of T_a, which then becomes the same as Eq. (3). This is the result that was to be demonstrated. It will be noted that this has been done without any restriction as to matched impedances at either J_1 or J_2, by employing the concepts of available power and available loss.

It may seem strange that a mismatch at J_1 affects the noise temperature of the network referred to the point J_2. Since Nyquist's theorem expresses noise voltage and power in relation to a dissipative circuit element, it might seem that the noise temperature should depend only on the dissipative loss of a circuit. But in the case of a four-terminal network it must be realized that the internally generated noise is delivered to two external loads which are effectively in series, i.e., to the circuit component to the left of J_1 as well as to the one to the right of J_2. Because of this series relationship, the impedance relationship at J_1 certainly would be expected to influence the power available at J_2, and definition of L_r as the available loss, in Eqs. (3), (34), and (34a), expresses this fact.

It is evident that Eqs. (3) and (34a) specify the noise temperature at the output terminals. The noise temperature referred to the input terminals would be obtained by multiplying the output temperature by L_r. T_r is evidently a spot-frequency noise temperature; i.e., if L_r is frequency dependent (as it may well be), so must also be T_r.

Receiver Noise Temperature

The evaluation of receiver noise temperature is ordinarily a laboratory measurement problem. The techniques are basically the same as those of noise-factor measurement (25). The relationship between noise-factor and noise-temperature measuring procedures is indicated by the equations that relate these quantities.

The IRE-defined effective input noise temperature, T_e is related to the receiver noise factor, $\overline{NF}$, by definition as follows:

$$T_e = (\overline{NF} - 1)\ T_o\ . \tag{4}$$

Actually the definition relates T_e only to the spot noise factor, so that T_e is implied to be a spot noise temperature. However, it is often used as an average noise temperature in system noise temperature equations, such as Eq. (2), although as previously mentioned it is then applicable only to single-response receivers. Its use as an average temperature is usually justified by the approximate equality of spot and average noise temperatures under ordinary conditions (see Appendix A).

Multiple-response receivers are characterized by either T_p or T_b as previously discussed and defined; see Eqs. (31) and (33). The relationships of the principal-response input noise temperature, T_p , and the broad-band input noise temperature, T_b, to the IRE-defined noise factor $\overline{NF}$, as derived in Appendix B, are

$$T_p = (\overline{NF} - \beta)\ T_o \tag{35}$$

$$\overline{NF} = \beta + T_p/T_o \tag{35a}$$

$$T_b = (\overline{NF}/\beta - 1)\ T_o \tag{36}$$

$$\overline{NF} = \beta(1 + T_b/T_o)\ . \tag{36a}$$

T_e, T_p, and T_b are all receiver input noise temperatures. It may occasionally be required or desired to characterize the receiver by an output noise temperature. In accordance with the referral principle, in the case of a single-response receiver the output temperature would be the input temperature T_e multiplied by the available gain of the receiver at the nominal response frequency. In the case of a multiple-response receiver it would be T_p or βT_b multiplied by the available gain at the nominal frequency of the principal response.

Cascade Formulas for System Noise Temperature

The noise temperature of the simplest form of cascade receiving system, consisting of an antenna, transmission line, and single-response receiver, has been given in Part I:

$$T_N = T_a/L_r + T_r + T_e \tag{2}$$

where T_a is the antenna noise temperature, L_r is the available-loss factor of the receiving transmission line, T_r is the effective output noise temperature of the transmission line, and T_e is the effective receiver input noise temperature, all evaluated at the nominal response frequency. That is, the quantities on the right-hand side are spot-frequency values, but are treated here as average values on the basis of the relations given in Appendix A.

A generalization of this equation for a receiver having s input responses can be made by applying the results of Eq. (28) to the antenna and transmission-line noise temperatures, and using the principal-response input noise temperature, T_p, in place of T_e:

$$T_N = T_p + \sum_{i=1}^{s} \left[G(f_i)/G(f_p)\right]\left[T_a(f_i)/L_r(f_i) + T_r(f_i)\right] \tag{37}$$

where $G(f_i)$ is the conversion gain of the receiver at the nominal frequency of the i^{th} response, f_p is the nominal frequency of the principal response, and $T_r(f_i)$ is given by

$$T_r = T_t\,(1 - 1/L_r) \tag{3}$$

with L_r evaluated at f_i.

Eq. (37) is not exact, but is a practical working formula based on assumptions that are usually justifiable, as discussed in connection with Eq. (28), and in Appendix A.

T_N in both Eqs. (2) and (37) is referred to the receiver input terminals, and assumes that the system consists solely of the three components: antenna, transmission line, and receiver. Any receiving system can be represented in this way, by considering the "transmission-line" as a catch-all containing any and all components interposed between the antenna and receiver (for example, a duplexer, hybrid junctions, tuning stubs, ferrite phase shifters, rotating joints, and the like). It is also possible, however, and it may in some cases be desirable, to consider the system as a cascade of many components. A formula for the noise temperature of such a system will be derived, and Eq. (2) will be shown to be a special case of this more general expression.

Figure 6 is a schematic diagram of such a system, in which n components precede the system-noise-temperature reference point, and m components follow it. Each component is characterized by a component noise temperature, representing its contribution of available noise power at its own input or output terminals. It is convenient to refer the noise temperatures of components <u>preceding</u> the reference point to their <u>output</u> terminals, and those of components <u>following</u> the reference point to their <u>input</u> terminals. This convention is assumed here. In the formula to be developed, the noise temperatures of components preceding the reference point will be designated by the subscript P, followed by a running index $j = 1, 2, 3, \ldots n$ denoting the position of the component in the cascade. The noise temperatures of components that follow the reference point will be designated by the subscript F, followed by a running index $k = 1, 2, 3, \ldots m$. Thus a noise temperature denoted by the subscripts Pj is defined by Eq. (24), while one with the subscripts Fk is defined by Eq. (25). Equations (26) and (28) may be applied when the assumptions made for Eq. (37) are valid.

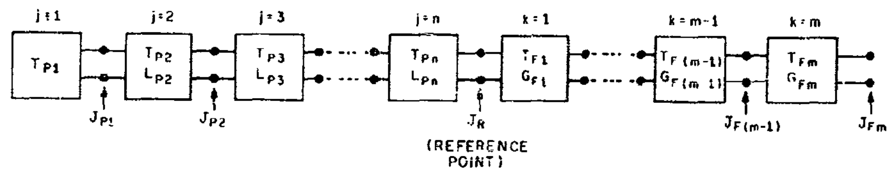

Fig. 6 - Receiving system with n components preceding the reference point and m components following it

Each component will be characterized by either an available gain or an available loss. Since the loss factor, L, is simply the reciprocal of the gain factor, G, either symbol could be used throughout the formula. It is convenient, however, or rather, customary, to use the gain factor to represent gains greater than unity, and loss factor to represent gains less than unity. In a general representation, it is of course not known whether a specific component will have a gain or a loss, in this sense. However, when the reference point is chosen as the receiver input terminals, it is most likely that preceding components will be characterized by loss and following components by gain. Hence, the formula will be written as though this were true. However, because of the equivalence of gain factor and reciprocal of loss factor, this does not represent a restriction on the generality of the formula. The same subscript notation will be used for the gain and loss factors as for the noise temperatures of the components.

As indicated in the discussion of the referral concept, each individual or component noise temperature is referred to the system-noise-temperature reference point, J_R in Fig. 6, by multiplying or dividing the component noise temperature by the net available power gain factor that applies to the portion of the system between the component-noise-temperature reference point and the system-noise-temperature reference point. The component noise temperature is multiplied by this gain factor (or divided by the net loss factor) if the component precedes the system reference point, and divided by the net gain factor (multiplied by net loss factor) if the component follows the system reference point. It will be apparent that these principles and conventions lead to the following formula for the system noise temperature, T_{NR} (with $R = n$):

$$\bar{T}_{NR} = \sum_{j=1}^{n-1} \bar{T}_{Pj} \left[\prod_{r=j+1}^{n} (1/L_{Pr})\right] + \bar{T}_{Pn} + \bar{T}_{F1} + \sum_{k=2}^{m} \bar{T}_{Fk} \left[\prod_{\ell=1}^{k-1} (1/G_{F\ell})\right]. \qquad (38)$$

Since this notation may not be universally understood, an example of expansion of this formula for the particular case $n = 4$ and $m = 3$ will be given:

$$T_{NR} = \bar{T}_{P1}/L_{P2}L_{P3}L_{P4} + \bar{T}_{P2}/L_{P3}L_{P4} + \bar{T}_{P3}/L_{P4} + \bar{T}_{P4} + \bar{T}_{F1} + \bar{T}_{F2}/G_{F1} + \bar{T}_{F3}/G_{F1}G_{F2} \,. \qquad (38a)$$

Moreover, Eqs. (2) and (37) are special cases of Eq. (38), where $n = 2$ and $m = 1$. T_a corresponds to $\bar{T}_{P1}$, T_r to $\bar{T}_{P2}$, L_r to L_{P2}, and T_e to $\bar{T}_{F1}$. In Eq. (37), $\bar{T}_{P1}$ and $\bar{T}_{P2}$ are represented by summations.

Special Rule Concerning Dissipative Loss in the Antenna

The antenna noise temperature, T_a, is ordinarily considered to account only for the noise power received by the antenna from external radiating sources. It does not (as here construed) take into account the thermal noise that will be generated if there is dissipative loss in the antenna. Although many types of antennas have negligible ohmic loss, this is by no means always the case. Some types of arrays, for example, have complicated transmission-line or waveguide feed structures. In some cases there may even be deliberately lossy elements in an antenna system, to terminate an element and avoid reflection, as in rhombic antennas and in waveguide slot arrays. Some types of phase-shifting devices (e.g., ferrites) may incur appreciable dissipative loss. Generally all of these types of lossy elements are regarded as part of the antenna system, and thus affect the noise power available at what would nominally be called the antenna terminals. In terms of Fig. 2, these lossy elements could be regarded as being within the block labeled antenna, and in Fig. 6 in the block for $j = 1$. But, if they are regarded as being thus located, the correct noise temperature to be assigned is not T_a, but rather an effective value given by

$$T_{a(eff)} = T_a/L_a + T_t(1 - 1/L_a) \tag{39}$$

where L_a is the ohmic loss of the antenna and T_t is its thermodynamic temperature.

This way of handling the matter seems less desirable (to the author) than an alternative method, in which the antenna is defined as comprising only the radiative (or, more properly in the present context, receptive) aspects of the physical structure generally regarded as the antenna. The "antenna terminals" may be thought of as located at the radiating surface of the antenna, beyond any conducting or dissipative portion of the structure. There may be no unique "point" in the antenna meeting this definition of the antenna terminals, but nevertheless the concept is a usable one. In terms of this definition of the antenna, an antenna temperature calculated in the general way described in Part I (e.g., a value given by Fig. 1) can be assigned as the noise temperature of the $j = 1$ block in Fig. 6. instead of going through the modification indicated by Eq. (39). Then, the $j = 2$ block can be used to represent the dissipative portion of the antenna, with $\bar{T}_{P2} = T_t(1 - 1/L_a)$ and, of course, $L_{P2} = L_a$. Or, in terms of the still simpler representation of Eq. (2), the dissipative portion of the antenna can be regarded as part of the transmission line; that is, L_a can be combined with the further losses of the transmission line system to arrive at a total loss factor, L_r, for use in Eq. (2). This is feasible when the thermodynamic temperatures of the lossy elements of antenna and of the transmission line are approximately the same.

To show the feasibility of this method, consider a system for which, in terms of Fig. 6, $n = 3$ and $m = 1$, where $i = 1$ represents the antenna in the special sense just defined, $j = 2$ represents the antenna losses, $j = 3$ represents the transmission line, and $k = 1$ represents the receiver. Let

$$\bar{T}_{P1} = T_a;\ \bar{T}_{P2} = T_{t2}(1-1/L_a),\ L_{P2} = L_a;\ \bar{T}_{P3} = T_{t3}(1-1/L_r);\ L_{P3} = L_r;\ \bar{T}_{F1} = T_e.$$

Then from Eq. (38),

$$T_N = T_a/L_aL_r + T_{t2}(1-1/L_a)/L_r + T_{t3}(1-1/L_r) + T_e \tag{40}$$

where T_{t2} and T_{t3} are the thermodynamic temperatures of the $j = 2$ and $j = 3$ blocks of Figs. 6. Now if $T_{t2} = T_{t3} = T_t$, this can be written

$$T_N = T_a/(L_aL_r) + T_t[1 - 1/(L_aL_r)] + T_e \tag{41}$$

which is seen to be the same as Eq. (2) except that L_r has been replaced by $L_a L_r$. Thus, in the convention here proposed, L_r in Eq. (2) may be interpreted as representing the total loss of the antenna and transmission line, subject only to the condition that the lossy elements of each are at the same thermodynamic temperatures. By an extension of this argument it is evident that any number of cascaded linear passive four-terminal networks can be lumped together in this way if their thermodynamic temperatures are all the same, or for practical purposes, nearly the same. This is the justification for employing the simpler Eqs. (2) or (37) instead of Eq. (38) when the "transmission line" may actually consist of a number of distinct cascaded elements.

The advantage of this way of handling antenna loss is that it permits the antenna noise temperature, in the sense that the term is generally understood, to be entered directly into either Eqs. (2), (37), or (38), instead of first being modified by Eq. (39). It is also compatible with the usual method of calculating received signal power for a given spatial power density at the antenna and the so-called receiving cross section of the antenna, in the sense that the value of received signal power thus obtained is also multiplied by $1/(L_a L_r)$ in order to obtain the available signal power at the receiver input terminals. In other words, this convention lends itself well to the problem of calculating signal-to-noise power ratio at the receiver terminals, as will be discussed subsequently in more detail. At the same time, it is not to be implied that this is the only way to handle the matter; either way is equally "correct," and it is really a matter of personal opinion as to which is the more logical and simple.

System Noise Temperature as a Figure of Merit

As has been indicated, the numerical value of the system noise temperature as here defined depends greatly upon the reference point chosen. Once this point has been specified, the system noise temperature becomes an index of the low-noise merit of the system, in the same sense that noise factor has for many years been an index of the low-noise merit of the receiver proper.* However, this value of system noise temperature cannot be used for comparison of one receiving system with another unless a special reference point is chosen, namely, the antenna terminals in the sense defined in the foregoing section. There is no other unique point within a receiving system that will serve this purpose, as may be demonstrated by considering simple hypothetical cases. At first it might be thought that the receiver input terminals would be such a point, but it is possible to show that two nonidentical systems having identical system noise temperatures referred to the antenna terminals could have different system temperatures referred to the receiver input terminals. In such a case it seems apparent that the former system temperature is the more valid index of the low-noise merit of the systems, because it can be shown that if the two systems have the same available signal power at the antenna terminals, they will also have the same signal-to-system-noise-power ratio at the output.

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*This role may now also be assumed by the receiver input noise temperature which, as others have pointed out, is a more appropriate and sensitive index for low-noise receivers since it directly corresponds to the receiver self-noise power level, while the noise factor corresponds to the level of receiver self-noise (referred to the input terminals) plus the noise of a standard-temperature source. For a discussion of this matter, see the correspondence of D. R. Rhodes and T. E. Talpey, "On the Definition of Noise Performance," IRE Proc. 49:376-377 (Jan. 1961). See also Ref. 28.

Because of this consideration, it might be contended that the term "system noise temperature" should imply only this reference point. However, the value calculated by Eq. (38) for any reference point is called system noise temperature in the sense that it represents the composite effect of all the individual noise sources in the system. In system performance calculations, where the need is to calculate a system signal-to-noise ratio, one reference point is as good as another, and the traditional one has been the receiver input terminals. Frankly, it was for this reason (tradition) that this reference point was chosen in writing Eq. (2) and making it the basis of a radar range calculation procedure (1,2); the input terminals could have been chosen just as well, but at the time, the idea of system noise temperature as a figure of merit for comparing different systems had not been considered (by the author).

The formula for the system input noise temperature, in the notation of Eq. (38), is

$$T_{NI} = \bar{T}_a + \bar{T}_{F1} + \sum_{k=2}^{m} \left\{ \bar{T}_{Fk} \left[\prod_{\ell=1}^{k-1} (1/G_{F\ell}) \right] \right\} \tag{42}$$

where T_a corresponds to T_{P1} in Eq. (38). The bar above T_a is used to denote the fact that an average temperature is meant, although ordinarily the spot temperature may be used (but in the case of a multiple-response receiver $\bar{T}_a$ would be given by an expression of the form of Eq. (28)). T_{F1} is the <u>input</u> average noise temperature of the first system component beyond the antenna, and would typically represent the lossy elements of the antenna if there is antenna dissipative loss; otherwise it would ordinarily be the effective <u>input</u> temperature of the first transmission-line component. Or, it may be regarded as the input temperature of the combination of all the transmission-line components, including lossy elements of the antenna, as in Eqs. (2) and (37). The equivalents of these three-component-system equations in terms of system input temperature are

$$T_{NI} = T_a + L_r (T_r + T_e) \tag{43}$$

and

$$T_{NI} = L_r(f_p)\, T_p + \sum_{i=1}^{s} \left[G(f_i)/G(f_p) \right] \left[T_a(f_i) + L_r(f_i)\, T_r(f_i) \right] \tag{44}$$

which represent the case $m = 2$ in Eq. (42), Eq. (43) being for a single-response system and Eq. (44) for a multiple-response system.

If the system input noise temperature is used in system performance (i.e., signal-to-noise ratio) calculations, care must be taken that the received signal power is referred to the same point. It must also be borne in mind that the "effective" antenna temperature given by Eq. (39), which includes the effect of antenna dissipative losses, cannot be used in Eqs. (42), (43), and (44); the antenna-temperature terms of these equations denote the temperature resulting solely from external radiating noise sources, typically represented by the values of Fig. 1.

Moreover, if the system input noise temperature is to be used for comparing the noise performance of different systems, identical assumptions must be made concerning the external noise environment. Therefore some generally accepted standards are needed, along the lines of the assumptions made in calculating the curve of Fig. 1, although it is not contended that these are the best possible assumptions for this purpose.

Alternatively, the low-noise merit of systems may be compared on the basis of a system input temperature given by Eq. (42) with the $\bar{T}_a$ term omitted. This temperature would be representative of the low-noise merit of the receiving system exclusive of the antenna. A suitable special name for this temperature might be "local-system input noise temperature." Of course this may also be regarded as the receiver input noise temperature, if the receiver is defined as the entire assemblage of components including transmission line. However, the purpose of this discussion is not to propose definitions or terminology, but rather to point out the technical factors that must be taken into account in rating systems.

SIGNAL-TO-NOISE-RATIO COMPUTATION

The ultimate objective in analysis of the system noise power is the calculation of signal-to-noise ratio. If (from either experiments or calculation based on information-theory considerations) the minimum observable or detectable signal-to-noise ratio is known, and if the spatial and spectral power densities of the signal impinging upon the receiving antenna for given radiated power density and propagation conditions can be calculated, the maximum useful range of the system can then be determined. A radar range equation incorporating the system noise temperature of Eq. (2) is given in Refs. 1 and 2, but it is not there shown how it is derived from basic principles. This will be done here in a brief fashion, utilizing the notation of Kerr (14).

Radio and Radar Equations

Initially it will be assumed that the spatial power density of the signal at the receiving antenna, designated S_r (watts per square meter), is known. The signal power captured by the receiving antenna is then

$$P_{ra} = S_r A_c \tag{45}$$

where A_c is the receiving-antenna capture area (square meters). This quantity is related to the receiving-antenna power gain by the well-known formula

$$A_c = \frac{G_r \lambda^2}{4\pi} . \tag{46}$$

Hence,

$$P_{ra} = \frac{S_r G_r \lambda^2}{4\pi} . \tag{47}$$

This is the available signal power at the antenna terminals. If the definition of A_c in Eq. (45) is taken to mean that P_{ra} is the power extracted by the antenna from the passing wave, G_r must be defined as the "pattern gain" of the antenna, also called "directivity" or "directive gain." It is the gain that is applicable to the definition of "the antenna" described in connection with accounting for antenna losses in system noise temperature calculation, in which antenna dissipative losses are separated from the purely receptive function of the antenna. The signal power calculated using this gain figure, P_{ra}, divided by the system input noise temperature of Eq. (42), gives the output signal-to-noise ratio.

If the system noise temperature is referred to any later point in the system, as it is for example in Eq. (2), then the available received signal power at this reference point is

$$P_r = \frac{S_r\ G_r\ \lambda^2}{4\ \pi\ L_r} \tag{48}$$

where L_r is the overall available loss (including that due to the antenna itself) between the antenna and the reference point, and the output signal-to-noise ratio is

$$S/N = \frac{P_r}{k\ T_N\ B} = \frac{S_r\ G_r\ \lambda^2}{4\ \pi\ L_r\ k\ T_N\ B}. \tag{49}$$

Although this expression was obtained as the ratio of the available powers, it is also the ratio of the actual powers, since both signal and noise are subject to the same mismatch loss. (It might at first be thought that this would not necessarily be true in the case of a multiple-response system, in which the mismatch ratio might be of different value at different responses; but this effect would be taken into account in the proper evaluation of the quantities in Eq. (37) in terms of available power and available gain.) This observation indicates the basic reason for employing the available-power concept.

It has been assumed that the system bandwidth is sufficient to pass the desired signal without appreciably changing its frequency spectrum. If the bandwidth is greater than this value, no bandwidth correction need be made to the resulting calculated value of signal-to-noise ratio. If the bandwidth is too small, a correction to P_{ra} or P_r is necessary, a reduction reflecting the effective signal-power loss due to the insufficient bandwidth. The effect of a larger-than-necessary bandwidth is automatically reflected in the signal-to-noise ratio through the presence of the factor B in the denominator of Eq. (49).

As is well known, for a pulse signal of specified waveform, there is an optimum width and shape of the receiver passband (1,2,9). The method of handling the effect of bandwidth on the signal-to-noise ratio in Refs. 1 and 2 is to assume that the optimum bandwidth has been used, and then to apply a bandwidth correction factor, C_B. Also, in the range equation, a quantity denoted "visibility factor" has been used in place of the signal-to-noise power ratio. It is defined as the ratio of the signal pulse <u>energy</u> to the noise power per unit bandwidth. This definition is advantageous in that it simplifies the range equation and allows a particularly desirable form of bandwidth-correction factor to be used. This highly specialized notation that has been developed for radar calculations is simply another way of expressing the results described here. The relationships of the visibility factor, signal-to-noise power ratio, and different bandwidth-correction factors are given in Ref. 1.

Equation (49) can be expanded for specific types of systems. If the received signal is from a radio transmitter of output power P_t watts, connected to an antenna of power gain G_t through a transmission line of loss factor L_t, well-known principles of wave propagation lead to the following expression for the spatial power density of the signal at a receiving point R meters distant:

$$S_r = \frac{P_t\ G_t\ F^2}{4\ \pi\ R^2\ L_t\ L_a} \tag{50}$$

where F is the pattern-propagation factor, as defined by Kerr (14), and L_a is a propagation-medium absorption loss factor. Inclusion of L_a here assumes that F does not take absorption losses into account. Values of L_a as a function of frequency for the case in which one terminal of the propagation path is ground-based (or at low altitude) are given in Ref. 3. The values given are in decibels and are for the radar case of two-way transit of the propagation path; therefore, these decibel values should be halved for application to a one-way communication circuit. The symbol L_a will be used indiscriminately in the equations that follow to apply to the total absorption loss; i.e., for one-way systems, the one-way loss will be meant, and for two-way systems the two-way loss will be meant. Therefore a complete expression for the signal-to-noise power ratio of a one-way radio system is

$$S/N = \frac{P_t\, G_t\, G_r\, \lambda^2\, F^2}{(4\,\pi)^2\, R^2\, L_t\, L_r\, L_a\, k\, T_N\, B_N}. \tag{51}$$

For the monostatic radar case, if the target has a monostatic radar cross section σ_m square meters, the corresponding equation is

$$S/N = \frac{P_t\, G_t\, G_r\, \sigma_m\, \lambda^2\, F^4}{(4\,\pi)^3\, R^4\, L_t\, L_r\, L_a\, k\, T_N\, B_N}. \tag{52}$$

These equations can of course be rearranged to express the range at which a specified signal-to-noise ratio will be obtained. Inserting the minimum useful (observable, audible, detectable, etc.) value of S/N then gives the maximum useful range of the system.

For a bistatic radar or communication system based on reflection from a satellite, if the distance from the transmitter to target or satellite is R_1, and from target or satellite to receiver R_2, and if the pattern-propagation factors for the two paths are F_1 and F_2, the equation is:

$$S/N = \frac{P_t\, G_t\, G_r\, \sigma_b\, \lambda^2\, F_1^2\, F_2^2}{(4\,\pi)^3\, R_1^2\, R_2^2\, L_t\, L_r\, L_a\, k\, T_N\, B_N} \tag{53}$$

where σ_b is the bistatic radar cross section of the target or the analogous quantity for the satellite. (If the latter is a perfectly reflecting sphere of radius r, the value to be used for σ_b is πr^2, assuming $r >> \lambda$).

These equations are for the signal-to-noise ratio on an instantaneous basis, and do not take into account any signal processing techniques of the type generally classified as integration or correlation which may result in an improved signal-to-noise ratio; i.e., these equations give the signal-to-noise ratio before processing of this kind.

Multiple-Signal-Channel Systems

In most radar and radio communication applications, one response of a multiple-response system is used for signal reception (the principal response), and the others are termed spurious responses. In the original draft of this report, only such systems were considered. It is of some interest, however, to consider how signal-to-noise ratio would be computed, utilizing the noise-temperature definitions assumed in this report, when signals are received in more than one channel. A more comprehensive discussion of this subject is given in Ref. 28 in terms of slightly different noise-temperature definitions.

If signal power is received via more than one input response (as in radiometry), the value of P_{ra} in Eq. (47) and of P_r in Eq. (48) should be multiplied by the ratio of (a) the total signal power output of the heterodyne converter within the passband of the succeeding portion of the system to (b) that which would have resulted if only the principal response accepted signal. If the signal power spatial density impinging upon the antenna is of uniform spectral density over the total range of signal acceptance and if the signals in the various responses are uncorrelated in rf phase, this ratio will simply be that of (a) the sum of the signal-power gains of all channels that accept signal to (b) the principal-response gain.

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APPENDIX A

PROOF OF EQUALITY OF SPOT AND AVERAGE NOISE TEMPERATURES UNDER CERTAIN CONDITIONS

In Part II it is stated that the average temperature over a passband and the spot temperature at the nominal frequency of the passband are equal if (a) the spot temperature has a constant value over the passband, or (b) the spot temperature varies linearly over the passband, the gain-frequency function is symmetrical about a center frequency, and the center frequency is the nominal frequency. Proofs of statements (a) and (b) for a single-response system, based on Eq. (24), follow. (For convenience, the subscript j will for the most part be omitted.)

Case (a). Let $T(f)$ in Eq. (24) be constant. Designating the nominal frequency of the passband by f_0, Eq. (24) then gives

$$\overline{T} = \frac{\int_0^\infty T(f)\,\mathcal{G}(f)\,df}{B_N\,\mathcal{G}(f_0)} = \frac{T(f_0)\int_0^\infty \mathcal{G}(f)\,df}{B_N\,\mathcal{G}(f_0)} \tag{A1}$$

and since, from Eq. (17)

$$B_N = \frac{\int_0^\infty \mathcal{G}(f)\,df}{\mathcal{G}(f_0)} \tag{A2}$$

this reduces to $\overline{T} = T(f_0)$, as was to be proved.

Case (b). Let $T(f)$ vary linearly, so that it can be expressed as

$$T(f) = T(f_0) + C\,(f - f_0) \tag{A3}$$

where C is the slope of $T(f)$. Also, let $\mathcal{G}(f)$ be symmetrical about f_0.

Now express these relationships in terms of a new variable $\xi = f - f_0$

$$T(\xi) = T(f_0) + C\,\xi \tag{A4}$$

$$\mathcal{G}(+\xi) = \mathcal{G}(-\xi)\ . \tag{A5}$$

Hence the numerator of Eq. (24) may be written

$$\int_0^\infty T(f)\,\mathcal{G}(f)\,df = \int_{-f_0}^\infty \left[T(f_0) + C\,\xi\right]\mathcal{G}(\xi)\,d\xi$$

$$= T(f_0)\int_{-f_0}^\infty \mathcal{G}(\xi)\,d\xi + C\int_{-f_0}^\infty \xi\,\mathcal{G}(\xi)\,d\xi\ . \tag{A6}$$

Now, since ξ is antisymmetric and $\mathcal{G}(\xi)$ is symmetric about $\xi = 0$, the second integral is zero, assuming that $\mathcal{G}(\xi) = 0$ for $\xi > f_0$. Also,

$$\int_{-f_0}^{\infty} \mathcal{G}(\hat{f})\, d\hat{f} = \int_{0}^{\infty} \mathcal{G}(f)\, df = B_N \mathcal{G}(f_0). \tag{A7}$$

Applying these results to Eq. (24) again gives $\bar{\bar{T}} = T(f_0)$, as was to be proved.

In the case of a multiple-response system, it is similarly demonstrable that $\bar{T}$ is the sum of the spot-temperature values in the individual passbands, with each such value weighted by the ratio of the gain for that passband to the gain in the principal-response passband, as indicated by the summations of Eqs. (28) and (29).

The effect of the factor b_{ji} of Eq. (28), or b_i of Eq. (28a), has been ignored in this proof — i.e., assumed equal to unity. This is equivalent to saying that $b_{ji} = 1$ is another condition that should in principle have been stated at the outset, but it is not a condition of any practical importance.

APPENDIX B

DERIVATION OF RELATIONS BETWEEN NOISE FACTOR AND NOISE TEMPERATURE

The IRE definition given in Ref. 25 of "noise factor at a specified input frequency" (spot noise factor), as applied to a multiple-response twoport transducer, is: "The ratio of (1) the total noise power per unit bandwidth at a corresponding output frequency available at the output port when the noise temperature of the input termination is standard (290°K) to (2) that portion of (1) engendered at the input frequency by the input termination."

As the ensuing discussion in Ref. 25 of measurement methods explains, the phrase "engendered at the input frequency" excludes from portion (2) of the definition the "noise from the input termination which appears in the output via ... image-frequency transformation." That is, only that noise from the input termination reaching the output via the principal response is included.

If the output noise power is observed within a narrow frequency band δf centered at the output frequency corresponding to the specified input frequency, f_I, the output noise power, portion (1) of the definition, consists of two parts. One is due to the input termination (at the standard noise temperature, $T_0 = 290°K$), and the other is due to the noise sources within the transducer itself. The sum of these two parts constitutes portion (1) of the noise-factor definition.

In terms of the quantity β defined by Eq. (31) of this report, the first of these parts is $\beta k T_0 \delta f G(f_I)$, where k is Boltzmann's constant and $G(f_I)$ is the transducer conversion gain at the input frequency f_I. The second part may be expressed in terms of the quantity defined as T_p, the principal-response input noise temperature, as $k T_p \delta f G(f_I)$.

Portion (2) of the definition is simply $k T_0 \delta f G(f_I)$. Therefore the noise factor $\overline{NF}$ is

$$\overline{NF} = \frac{\beta k T_0 \delta f G(f_I) + k T_p \delta f G(f_I)}{k T_0 \delta f G(f_I)} \tag{B1}$$

or

$$\overline{NF} = \beta + T_p/T_0 \tag{35a}$$

which, as the number indicates, is Eq. (35a) of the report. From this, Eq. (35) is obtained:

$$T_p = (\overline{NF} - \beta) T_0 \tag{35}$$

and since by definition

$$T_e = (\overline{NF} - 1) T_0 \tag{4}$$

it is immediately deducible that

$$T_e = T_p + (\beta - 1) T_0 \tag{33a}$$

which conforms to the statement that T_e expresses the intrinsic transducer noise plus the contribution of the standard-temperature input termination via the spurious responses.

It may be helpful for understanding the significance of T_e in the multiple-response case to write Eq. (4) in the following way:

$$\overline{NF} = 1 + \frac{T_e}{T_0} = \frac{T_0 + T_e}{T_0} \tag{B2}$$

Multiplying both numerator and denominator by $k\,\delta f\,G(f_I)$ results in an equation analogous to Eq. (B1):

$$\overline{NF} = \frac{k\,T_0\,\delta f\,G(f_I) + k\,T_e\,\delta f\,G(f_I)}{k\,T_0\,\delta f\,G(f_I)}\,. \tag{B3}$$

Comparing Eq. (B3) with Eq. (B1) it is apparent that the portion of the input-termination noise power due to the spurious responses, which is ([illegible] $- 1)\,k\,T_0\,\delta f\,G(f_I)$, has been taken out of the first term of the numerator. Therefore, this noise power must be included in the definition of T_e which appears in the second term of the numerator, in order that the entire numerator shall represent the total output noise power.

The representation of the noise factor by Eq. (B1) is based on the author's interpretation of the stated definition of noise factor, and is possibly controversial, although a number of well-qualified engineers concur that it is a correct interpretation. Although nobody whose opinion was sought disagreed with the interpretation, there were some who were not sure that it is correct. As a matter of fact, the formula given in the IRE Standards paper (Ref. 25) for the noise factor of a cascade system in terms of the component noise factors implies that the factor [illegible] should be omitted from the numerator of Eq. (B1), which would in turn make the definition of T_e conform to that of T_p. However, the verbal statement of the noise-factor definition, and the measurement method described for the case of a heterodyne transducer, argue strongly that the factor [illegible] belongs in Eq. (B1).

Table B1 is helpful in numerical evaluation of the quantity T_e when $\overline{NF}$ is given, or vice versa. T_p may be determined from T_e, of course, by subtracting from it the quantity ([illegible] $- 1)\,T_0$.

The table also provides for numerical evaluation of the quantity T_r, receiving-transmission-line output noise temperature, given the value of the loss factor L_r expressed either as a power ratio or as a decibel loss.

Table B1
Decibels, Power Ratios, and Effective Noise Temperatures

1. The first three columns are a standard table of decibel values and corresponding power ratios. The second column is the ratio corresponding to a positive decibel value, and the third corresponds to a negative decibel value.

If the given quantity is a positive or negative decibel value greater than 10, the first three columns may be used as follows. From the absolute value of the given decibel figure, subtract 10n, where n is a positive integer such that the resulting remainder is a positive number less than 10. In the table, find the power ratio corresponding to this decibel remainder. If the original decibel value is positive, multiply this power ratio by 10^n; if negative, by 10^{-n}.

If the given quantity is a power ratio greater than 10 or less than 0.1, multiply it by 10^{-n} or 10^n respectively, where n is the integer required to bring the resulting power ratio within the range of values found in the second and third columns. Look up the decibel value (corresponding to the resulting power ratio) and increase its absolute value by 10n.

2. If the second column represents a loss ratio, L, the third column is its reciprocal, which occurs in the formula for the effective noise temperature of a lossy receiving transmission line or propagation medium:

$$T_r = T_t\left(1 - \frac{1}{L}\right)$$

where T_r is the effective noise temperature and T_t is the thermal temperature (Kelvin) of the line or medium. The fourth column is the resulting noise temperature, calculated from this equation, assuming $T_t = 290°K$. If in the actual case T_t has some other value, multiply the values of T_r by $T_t/290$.

3. The second column also represents the receiver noise factor, $\overline{NF}$, expressed as a power ratio, corresponding to decibel noise-factor values represented by the first column. The fifth column is the corresponding effective receiver noise temperature, $T_e = (\overline{NF} - 1)T_0$. ($T_0 = 290°K$).

Decibels $\pm$	Power Ratios + (NF)	Power Ratios − ($1/L_r$)	T_r °Kelvin	T_e °Kelvin
0	1.0000	1.0000	0.00	0.00
0.01	1.0023	.9977	0.67	0.67
0.02	1.0046	.9954	1.33	1.33
0.03	1.0069	.9931	2.00	2.00
0.04	1.0093	.9908	2.67	2.70
0.05	1.0116	.9886	3.31	3.36
0.06	1.0139	.9863	3.97	4.03
0.07	1.0162	.9840	4.64	4.70
0.08	1.0186	.9818	5.28	5.39
0.09	1.0209	.9795	5.95	6.05
0.10	1.0233	.9772	6.61	6.76
0.15	1.0351	.9661	9.83	10.2
0.20	1.0471	.9550	13.1	13.7
0.25	1.0593	.9441	16.2	17.2
0.30	1.0715	.9333	19.3	20.7
0.35	1.0839	.9226	22.4	24.3
0.40	1.0965	.9120	25.5	28.0
0.45	1.1092	.9016	28.5	31.7
0.50	1.1220	.8913	31.5	35.4
0.55	1.1350	.8810	34.5	39.2
0.60	1.1482	.8710	37.4	43.0
0.65	1.1614	.8610	40.3	46.8
0.70	1.1749	.8511	43.2	50.7
0.75	1.1885	.8414	46.0	54.7
0.80	1.2023	.8318	48.8	58.7
0.85	1.2162	.8222	51.6	62.7
0.90	1.2303	.8128	54.3	66.8
0.95	1.2445	.8035	57.0	70.9
1.00	1.2589	.7943	59.7	75.1
1.1	1.288	.7762	64.9	83.5
1.2	1.318	.7586	70.0	92.2
1.3	1.349	.7413	75.0	101
1.4	1.380	.7244	79.9	110
1.5	1.413	.7079	84.7	120
1.6	1.445	.6918	89.4	129
1.7	1.479	.6761	93.9	139
1.8	1.514	.6607	98.4	149
1.9	1.549	.6457	103	159
2.0	1.585	.6310	107	170
2.1	1.622	.6166	111	180
2.2	1.660	.6026	115	191
2.3	1.698	.5888	119	202
2.4	1.738	.5754	123	214
2.5	1.778	.5623	127	226
2.6	1.820	.5495	131	238
2.7	1.862	.5370	134	250
2.8	1.905	.5248	138	262
2.9	1.950	.5129	141	276
3.0	1.995	.5012	145	289
3.1	2.042	.4898	148	302
3.2	2.089	.4786	151	316
3.3	2.138	.4677	154	330
3.4	2.188	.4571	157	345
3.5	2.239	.4467	160	359
3.6	2.291	.4365	163	374
3.7	2.344	.4266	166	390
3.8	2.399	.4169	169	406
3.9	2.455	.4074	172	422
4.0	2.512	.3981	175	438
4.1	2.570	.3890	177	455
4.2	2.630	.3802	180	473
4.3	2.692	.3715	182	491
4.4	2.754	.3631	185	509
4.5	2.818	.3548	187	527
4.6	2.884	.3467	189	546
4.7	2.951	.3388	192	566
4.8	3.020	.3311	194	586
4.9	3.090	.3236	196	606
5.0	3.162	.3162	198	627
5.1	3.236	.3090	200	648
5.2	3.311	.3020	202	670
5.3	3.388	.2951	204	693
5.4	3.467	.2884	206	715
5.5	3.548	.2818	208	739
5.6	3.631	.2754	210	763
5.7	3.715	.2692	212	787
5.8	3.802	.2630	214	813
5.9	3.890	.2570	215	838
6.0	3.981	.2512	217	864
6.1	4.074	.2455	219	891
6.2	4.169	.2399	220	919
6.3	4.266	.2344	222	947
6.4	4.365	.2291	224	976
6.5	4.467	.2239	225	1005
6.6	4.571	.2188	227	1036
6.7	4.677	.2138	228	1066
6.8	4.786	.2089	229	1098
6.9	4.898	.2042	231	1130
7.0	5.012	.1995	232	1163
7.1	5.129	.1950	233	1197
7.2	5.248	.1905	235	1232
7.3	5.370	.1862	236	1267
7.4	5.495	.1820	237	1304
7.5	5.623	.1778	238	1341
7.6	5.754	.1738	240	1379
7.7	5.888	.1698	241	1418
7.8	6.026	.1660	242	1458
7.9	6.166	.1622	243	1498
8.0	6.310	.1585	244	1540
8.1	6.457	.1549	245	1583
8.2	6.607	.1514	246	1626
8.3	6.761	.1479	247	1671
8.4	6.918	.1445	248	1716
8.5	7.079	.1413	249	1763
8.6	7.244	.1380	250	1811
8.7	7.413	.1349	251	1860
8.8	7.586	.1318	252	1910
8.9	7.762	.1288	253	1961
9.0	7.943	.1259	253	2013
9.1	8.128	.1230	254	2067
9.2	8.318	.1202	255	2122
9.3	8.511	.1175	256	2178
9.4	8.710	.1148	257	2236
9.5	8.913	.1122	257	2295
9.6	9.120	.1096	258	2355
9.7	9.333	.1072	259	2417
9.8	9.550	.1047	260	2480
9.9	9.772	.1023	260	2544
10.0	10.000	.1000	261	2610

Temperature conversion relations: $T_{Kelvin} = 273.16 + T_{Centigrade} = 255.38 + (5/9)\,T_{Fahrenheit}$

290° Kelvin = 16.84° Centigrade = 62.32° Fahrenheit

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of cascaded components, referred to an arbitrary point within the system. The need for definition of both the spot (frequency-dependent) noise temperature and the average temperature over a passband is pointed out, and also the need for definition of a transducer noise temperature that represents only the intrinsic transducer noise. The use of system noise temperature for comparing the low-noise merit of different systems is discussed. It is pointed out that for this purpose the system temperature must be referred to the system input terminals, and that these terminals must be defined to precede all system elements that result in dissipative loss, including loss that may occur in the antenna structure. Moreover, if the antenna is included as part of a system being thus rated, some standard or convention as to the noise environment (such as the assumptions made in calculating the curve in Part I) is needed. The calculation of received signal power for various types of systems (one-way radio, monostatic and bistatic radar, satellite reflection communication) is briefly reviewed, to show how the system noise temperature may be used for signal-to-noise-ratio calculation. The case in which signal power may be simultaneously received via more than one input response channel of a multiple-response receiver (as in radiometry) is briefly considered. The report is basically oriented to the problem of radar maximum range calculation, but has application to radio receiving systems in general.

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